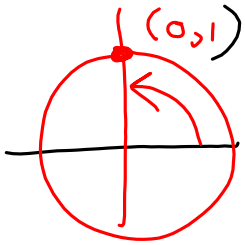
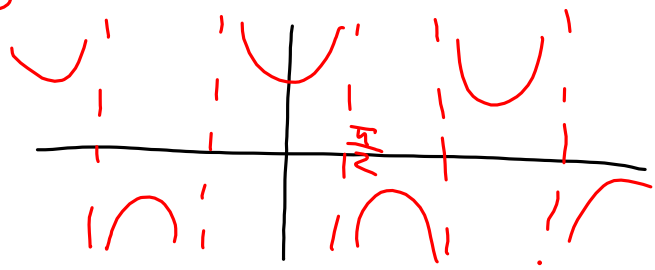
20. $\lim_{x \to \frac{\pi}{2}} \sec x$ does not exist



$\sec \frac{\pi}{2} = \frac{1}{\cos \pi/2} = \frac{1}{0}$ undefined

$\lim_{x \to \frac{\pi}{2}^+} \sec x = -\infty$



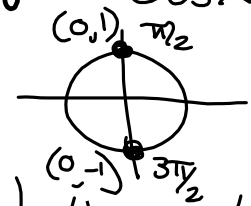
52. $f(x) = \tan \frac{\pi x}{2}$

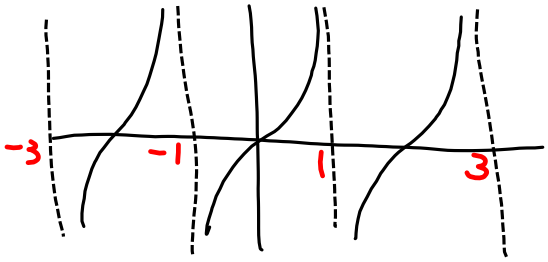
$y = \tan x = \frac{\sin x}{\cos x}$

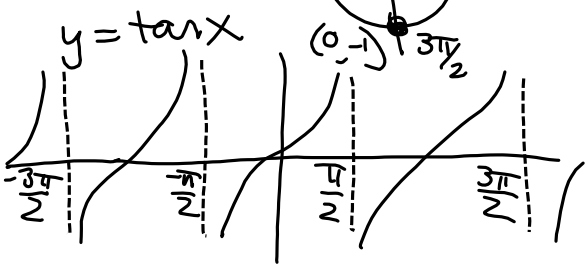
V.A.'s when $\cos x = 0$

discuss the (dis)continuity

$\tan \frac{\pi x}{2}$ period: $\frac{\pi}{\pi/2} = \frac{\pi}{1} \cdot \frac{2}{\pi} = 2$







f(x) has non-removable discontinuities at all odd integers

f is continuous on all intervals of the form $(2n-1, 2n+1)$, $n \in \mathbb{Z}$

$$f(x)=\begin{cases}\dfrac{|x-3|}{3-x}, & |x-3|>5 \\ x^2-3, & -2\le x\le 8\end{cases}$$

$|x-3|>5$: $x-3>5$ or $x-3<-5$; $x>8$ or $x<-2$

$\dfrac{|-2-3|}{3-(-2)}=1 \stackrel{?}{=} 1=(-2)^2-3$

$\dfrac{|8-3|}{3-8}=-1 \neq 61=(8)^2-3$

f has a non-removable discontinuity @ x=8

f is continuous on $(-\infty, 8] \cup (8, \infty)$

Discuss the(dis)continuity of $f(g(x))$.

$f(x)=\dfrac{1}{\sqrt{x}}$, $(0,\infty)$; $g(x)=x-1$, $(-\infty,\infty)$

$(f\circ g)(x)=\dfrac{1}{\sqrt{x-1}}$

fog is continuous on its domain $\{x \mid x-1>0\}$, $x>1$, $(1,\infty)$

$f(x)=\sin x$, $(-\infty,\infty)$; $g(x)=x^2$, $(-\infty,\infty)$

$(f\circ g)(x)=\sin(x^2)$ continuous on $(-\infty,\infty)$