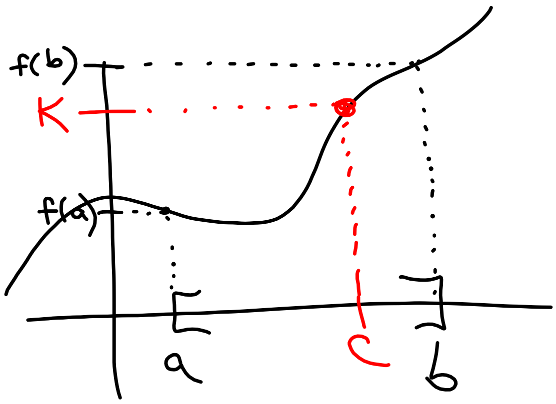
<u>Intermediate Value Theorem</u>

If *f* is continuous on the closed interval [*a*,*b*] and *k* is any number between *f(a)* and *f(b)*, then there is at least one number *c* in [*a*,*b*] such that *f(c)=k*.



Does the IVT guarantee a zero in the given interval?

76. $f(x) = x^3 + 3x - 2$, $[0,1]$

84. $f(x) = x^2 - 6x + 8$; $[0, 3]$; $f(c) = 0$

86. $f(x) = \frac{x^2+x}{x-1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$

1.5 **Infinite Limits**

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x \to c} f(x) = \infty$ *and* $\lim_{x \to c} g(x) = L$

1. $\lim_{x \to c} [f(x) \pm g(x)] = \infty$

2. $\lim_{x \to c} [f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

$\frac{2}{1}, \frac{2}{10}, \frac{2}{100}, \frac{2}{1000}, \frac{2}{1,000,000}$

Find the vertical asymptotes (if any).

1. $f(x) = \frac{-4x}{x^2+4}$

no vertical asymptotes

$x^2+4=0$

$x^2=-4$

$x=\pm 2i$

2. $h(x) = \frac{x^2-4}{x^3+2x^2+x+2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2+1)}$

does not have any real zeros

no vertical asymptotes

$\underbrace{x^3+2x^2}+\underbrace{x+2}=0$

$x^2(x+2)+1(x+2)$

$(x+2)(x^2+1)$

3. $g(\theta) = \frac{\tan\theta}{\theta}$

V.A.'s wherever $\cos\theta = 0$

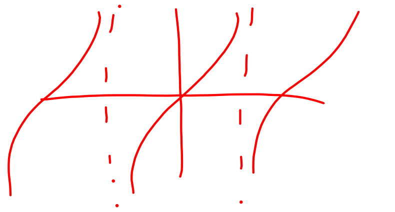
$\ldots, -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\lim_{\theta\to 0} \frac{\tan\theta}{\theta}$

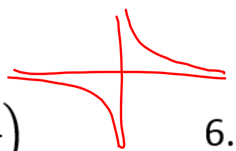
$= \lim_{\theta\to 0} \frac{\sin\theta}{\theta\cos\theta}$

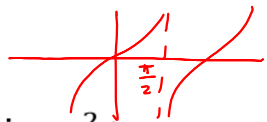
$= \left(\lim_{\theta\to 0} \frac{\sin\theta}{\theta}\right)\left(\lim_{\theta\to 0} \frac{1}{\cos\theta}\right)$

$= 1 \cdot 1 = 1$



Find the limits.





4. $\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right)$

$= \lim_{x\to 0^-} x^2 - \lim_{x\to 0^-} \frac{1}{x}$

$= 0 - (-\infty) = \boxed{\infty}$

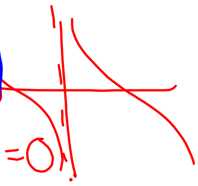
6. $\lim_{x\to \frac{1}{2}} x^2 \tan \pi x$

$= \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}} \tan \pi x\right)$

$= \frac{1}{4} \cdot \lim_{x\to\frac{\pi}{2}} \tan x = \frac{1}{4} \cdot \text{DNE}$

Does not exist

5. $\lim_{x\to 0} \frac{x+2}{\cot x} = \boxed{0}$



$\lim_{x\to 0^-} \frac{x+2}{\cot x} = \frac{2}{-\infty} = 0$

$\lim_{x\to 0^+} \frac{x+2}{\cot x} = \frac{2}{\infty} = 0$

7. $\lim_{x\to 3^+} \sec \frac{\pi x}{6}$

$= \lim_{x\to \frac{3\pi}{6}^+} \sec x$

$= \lim_{x\to\frac{\pi}{2}^+} \sec x = \boxed{-\infty}$