$$f(x) = \sqrt[3]{\csc(4x)} = \left[\csc 4x\right]^{1/3}$$

$$\boxed{f'(x) = \frac{1}{3}\left[\csc 4x\right]^{-2/3} \cdot \left(-\csc 4x \cot 4x\right) \cdot 4}$$

$$= \frac{-4\csc 4x \cot 4x}{3(\csc 4x)^{2/3}} = \frac{-4(\csc 4x)^{1/3}\cot 4x}{3} =$$

$$= \frac{-4\cos 4x}{3(\sin 4x)^{1/3}\sin 4x} = \frac{-4\cos 4x}{3(\sin 4x)^{4/3}} = \frac{-4\cos 4x}{3\sqrt[3]{\sin^4 4x}}$$

$$f(x) = \frac{\sin 2x}{x^3} = (\sin 2x)(x^{-3})$$

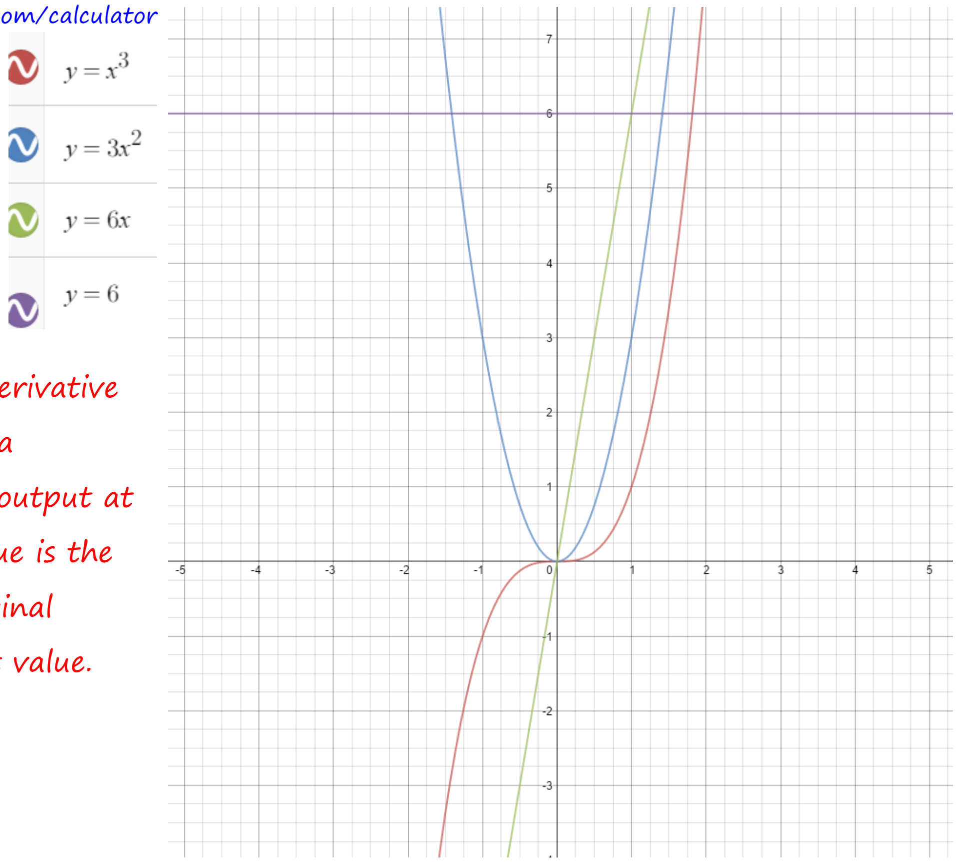
$$f'(x) = (\sin 2x)'(x^{-3}) + (\sin 2x)(x^{-3})'$$

$$= \boxed{(2\cos 2x)(x^{-3}) + (\sin 2x)(-3x^{-4})}$$

$$= \frac{2\cos 2x}{x^3} - \frac{3\sin 2x}{x^4}$$

$$= \frac{2x\cos 2x - 3\sin 2x}{x^4}$$

https://www.desmos.com/calculator




*Note that the derivative of a function is a function whose output at a particular value is the slope of the original function at that value.*

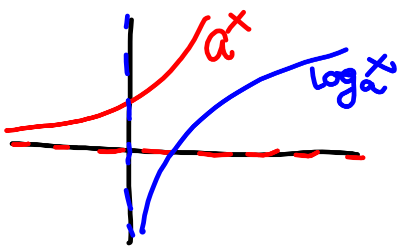
## Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \quad \Leftrightarrow \quad 2^3 = 8$

$\log_a b = c \quad \Leftrightarrow \quad a^c = b$




$y = 2^x$

$x$ = the power to which we raise 2 to get $y$

= the # of times we multiply 2 by itself to get $y$

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = 2^x \ln 2$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \ln 2}$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

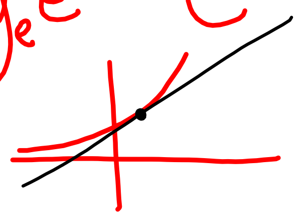
$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \cdot \ln a} \cdot f'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$$

$$\left[e^x\right]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$\boxed{\begin{array}{l} \log_a a = 1 \\ \ln e = \log_e e \end{array}}$$

$$\left[e^x\right]' = e^x$$



$$\left[\ln x\right]' = \frac{1}{x \ln e} = \frac{1}{x}$$

$$\ln x = \log_e x$$

*Since the derivative of $e^x$ is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.*

$$f(x) = \ln\left[\sin\left(5x^3+2x\right)\right]$$

$$f'(x) = \boxed{\frac{1}{\sin(5x^3+2x)} \cdot \left[\cos\left(5x^3+2x\right)\right] \cdot \left(15x^2+2\right)}$$

$$= \frac{(15x^2+2)\cos(5x^3+2x)}{\sin(5x^3+2x)}$$

$$= (15x^2+2)\cot(5x^3+2x)$$

$$= \frac{15x^2+2}{\tan(5x^3+2x)}$$

$$f(x) = (\sec x)(5^{\sin x}) \qquad \left[a^{f(x)}\right]' = a^{f(x)} \ln a \cdot f'(x)$$

$$f'(x) = (\sec x)' \cdot (5^{\sin x}) + (\sec x)(5^{\sin x})'$$

$$= \boxed{(\sec x \tan x)(5^{\sin x}) + (\sec x)(5^{\sin x} \ln 5 \cdot \cos x)}$$

$$= \boxed{(\sec x \tan x)(5^{\sin x}) + 5^{\sin x} \ln 5}$$

$$5^{\sin x} = \frac{1}{5^{-\sin x}} \qquad a^n = \frac{1}{a^{-n}}$$

$$f(x) = \frac{(x^2 \ln x)}{(\sin x)}$$

$$\boxed{f'(x) = \frac{(\sin x)\left(2x\ln x + x^2 \cdot \frac{1}{x}\right) - (x^2 \ln x)(\cos x)}{\sin^2 x}}$$

OR $f(x) = (x^2 \ln x)\csc x$

$$f'(x) = \left(2x\ln x + x^2 \cdot \frac{1}{x}\right) \cdot \csc x + x^2 \ln x(-\csc x \cot x)$$