$$f(x)=\sqrt[3]{\sin^2(\ln(4x^9))} = \left[(\sin(\ln(4x^9)))^2\right]^{1/3}$$
$$= [\sin(\ln(4x^9))]^{2/3}$$

$$\boxed{(x^m)^n = x^{mn}}$$

$$f'(x) = \frac{2}{3}[\sin(\ln(4x^9))]^{-1/3} \cdot \cos(\ln(4x^9)) \cdot \frac{1}{4x^9} \cdot 36x^8$$

$$f(x) = 5^{\sqrt[3]{4\log_2(3x^2-4x)}} = 5^{(4\log_2(3x^2-4x))^{1/3}}$$

$$f'(x) = 5^{\sqrt[3]{4\log_2(3x^2-4x)}} \cdot \ln 5 \cdot \frac{1}{3}(4\log_2(3x^2-4x))^{-2/3} \cdot$$
$$\cdot 4 \cdot \frac{1}{(3x^2-4x)\ln 2} \cdot (6x-4)$$

2.4 The Chain Rule, cont.

18. $f(x) = -3\sqrt[4]{2-9x}$ $= -3(2-9x)^{1/4}$

$$f'(x) = \boxed{-\frac{3}{4}(2-9x)^{-3/4} \cdot (-9)} = \frac{27}{4\sqrt[4]{(2-9x)^3}}$$

32. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3+2}\right) \cdot \frac{(t^3+2)(2t)-(t^2)(3t^2)}{(t^3+2)^2}$$

50. $h(x) = \sec x^2$ $= \sec(x^2)$

$$h'(x) = [\sec(x^2)\tan(x^2)] \cdot 2x$$
$$= 2x\sec x^2 \tan x^2$$

60. $g(t) = 5\cos^2 \pi t$ $= 5[\cos \pi t]^2$

$g'(t) = 10\cos\pi t \cdot (-\sin\pi t) \cdot \pi$

$= \boxed{-10\pi\cos\pi t\sin\pi t} = \boxed{-5\pi\sin 2\pi t}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad -5\cdot 2\sin(\pi t)\cos(\pi t)$

66. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$

$= \sin(x^{1/3}) + (\sin x)^{1/3}$

$y' = [\cos(x^{1/3})] \cdot \frac{1}{3}x^{-2/3} + \frac{1}{3}(\sin x)^{-2/3} \cdot \cos x$

5.4

$[e^{f(x)}]' = e^{f(x)} \cdot f'(x)$

46. $g(t) = e^{-3/t^2}$ $= e^{-3t^{-2}}$

$g'(t) = e^{-3t^{-2}} \cdot 6t^{-3}$

$f(x) = e^x$

$[f(g(x))]' = [e^{g(x)}]'$

$f'(g(x)) \cdot g'(x) = e^{g(x)} \cdot g'(x)$

$\log_a MN = \log_a M + \log_a N$

$\log_a \frac{M}{N} = \log_a M - \log_a N$

$\log_a (x^p) = p\log_a x$

48. $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$

$= \ln(1+e^x) - \ln(1-e^x)$

$y' = \frac{1}{1+e^x} \cdot e^x - \frac{1}{1-e^x} \cdot (-e^x)$

$= \boxed{\frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}}$

58. $y = \ln e^x$ $= x$

$y' = 1$

$y' = \frac{1}{e^x} \cdot e^x = 1$

$\log_a a^x = x; \; a^{\log_a x} = x$

<u>5.5</u>

46. $f(t) = \frac{3^{2t}}{t} = (3^{2t})(t^{-1})$

$f'(t) = (3^{2t} \cdot \ln 3 \cdot 2)(t^{-1}) + (3^{2t})(-t^{-2})$

54. $y = \log_{10} \frac{x^2-1}{x} = \log_{10}(x^2-1) - \log_{10} x$

$y' = \frac{1}{(x^2-1)\ln 10} \cdot (2x-0) - \frac{1}{x \ln 10}$

$y' = \frac{2x}{(x^2-1)\ln 10} - \frac{1}{x \ln 10}$

$[x^n]' = nx^{n-1}$

$[cf(x)]' = c\,f'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$[e^x]' = e^x$

$[a^x]' = a^x \ln a$

$[\ln x]' = \frac{1}{x}$

$[\log_a x]' = \frac{1}{x \ln a}$

$[\sin x]' = \cos x$

$[\cos x]' = -\sin x$

$[\tan x]' = \sec^2 x$

$[\cot x]' = -\csc^2 x$

$[\sec x]' = \sec x \tan x$

$[\csc x]' = -\csc x \cot x$

$[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$

$[\arctan x]' = \frac{1}{1+x^2}$

$[\text{arcsec}\, x]' = \frac{1}{|x|\sqrt{x^2-1}}$

$[\arccos x]' = \frac{-1}{\sqrt{1-x^2}}$

$[\text{arccot}\, x]' = \frac{-1}{1+x^2}$

$[\text{arccsc}\, x]' = \frac{-1}{|x|\sqrt{x^2-1}}$

$$\arcsin x = \sin^{-1} x \neq \frac{1}{\sin x}$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

<u>5.8</u>

44. $f(x) = \operatorname{arcsec} 2x$

$$f'(x) = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2$$

$$= \frac{1}{|x|\sqrt{4x^2-1}}$$

48. $h(x) = x^2 \arctan x$

52. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

$$\boxed{\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arccsc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$$