What happens if...

explicit
$y = f(x)$

$$x^2 y + y^2 x = -2$$

how to find y'?

2.5 Implicit Differentiation

$y = f(x)$

y is a function of x

$$\frac{d}{dx}[x] = 1 \quad ; \quad \frac{d}{dx}[y] = y' = \frac{dy}{dx}$$

$$y = e^{-x}$$
$$y' = e^{-x} \cdot (-1)$$

$$\frac{d}{dx}\left[e^{y}\right] = e^{y} \cdot \frac{d}{dx} y$$
$$= e^{y} \cdot y'$$

6. $x^2y + y^2x = -2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}\left[-2\right]$$

$$\frac{d}{dx}\left[x^2y\right] + \frac{d}{dx}\left[y^2x\right] = 0$$

$$\frac{d}{dx}\left[x^2\right]\cdot y + x^2\cdot\frac{d}{dx}\left[y\right] + \frac{d}{dx}\left[y^2\right]\cdot x + y^2\cdot\frac{d}{dx}\left[x\right] = 0$$

$$2xy + x^2\cdot y' + 2y\cdot y'\cdot x + y^2\cdot 1 = 0$$

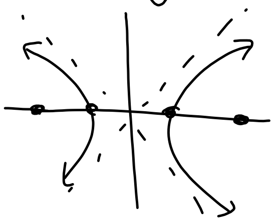
$$x^2y' + 2xyy' = -y^2 - 2xy$$

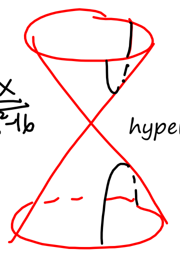
$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16 \rightarrow x^2 - 16 = y^2$

$\pm\sqrt{x^2-16} = y \Rightarrow y' = \frac{\pm 2x}{2\sqrt{x^2-16}}$





hyperbola

$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2y\cdot y' = 0$$

$$\frac{2x}{2y} = \frac{2yy'}{2y}$$

$$\boxed{y' = \frac{x}{y}}$$

8. $\sqrt{xy} = x - 2y$

$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}[x-2y]$

$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}[xy] = 1 - 2y'$

$\cancel{2\sqrt{xy}} \cdot \cancel{\frac{1}{2\sqrt{xy}}} \cdot [1 \cdot y + x \cdot y'] = (1 - 2y') \cdot 2\sqrt{xy}$

$y + xy' = 2\sqrt{xy} - 4y'\sqrt{xy}$

$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$

$y'(x + 4\sqrt{xy}) = 2\sqrt{xy} - y$

$\boxed{y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}}$

10. $2\sin x \cos y = 1$

$\frac{d}{dx}[\sin x \cos y] = \frac{d}{dx}\left[\frac{1}{2}\right]$

$(\sin x)'\cos y + \sin x(\cos y)' = 0$

$\cos x \cos y + \sin x(-y'\sin y) = 0$

$\cos x \cos y = y'\sin x \sin y$

$\frac{\cos x \cos y}{\sin x \sin y} = y'$

$\boxed{\cot x \cot y = y'}$

or divide both sides by $2\sin x$ $\rightarrow$ $(\cos y)' = \left(\frac{1}{2}\csc x\right)'$

$-y'\sin y = -\frac{1}{2}\csc x \cot x$

$\boxed{y' = \frac{\csc x \cot x}{2\sin y}}$

12. $(\sin \pi x + \cos \pi y)^2 = 2$

$\Rightarrow \sin \pi x + \cos \pi y = \pm\sqrt{2}$

$\Rightarrow \sin \pi x + \cos \pi y = \sqrt{2}$ or $\sin \pi x + \cos \pi y = -\sqrt{2}$

$$\frac{d}{dx}[\sin \pi x + \cos \pi y] = 0$$

$$\pi \cos \pi x - \pi y' \sin \pi y = 0$$

$$\pi \cos \pi x = \pi y' \sin \pi y$$

$$\frac{\cancel{\pi} \cos \pi x}{\cancel{\pi} \sin \pi y} = y'$$

$$\boxed{\frac{\cos \pi x}{\sin \pi y} = y'}$$

16. $x = \sec\left(\frac{1}{y}\right)$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec\left(y^{-1}\right)\right]$$

$$1 = \left(\sec\tfrac{1}{y}\tan\tfrac{1}{y} \cdot -y^{-2}\right) \cdot y'$$

$$\frac{1}{\left(-y^{-2}\sec\tfrac{1}{y}\tan\tfrac{1}{y}\right)} = y'$$

$$\frac{-y^2}{\sec\tfrac{1}{y}\tan\tfrac{1}{y}} = y'$$

$$\boxed{-y^2\cos\tfrac{1}{y}\cot\tfrac{1}{y} = y'}$$

$$\cancel{x = \frac{1}{\cos\frac{1}{y}}}$$

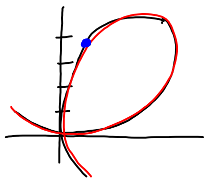
$$\cancel{\cos\frac{1}{y} = \frac{1}{x}}$$

$$\boxed{\frac{1}{x^{-n}} = \frac{x^n}{1}}$$

32. Folium of Descartes

$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$$\frac{d}{dx}\left[x^3+y^3\right] = \frac{d}{dx}\left[6xy\right]$$

$$3x^2+3y^2y' = 6y+6xy'$$

$$3y^2y'-6xy' = 6y-3x^2$$

$$3y'\left(y^2-2x\right) = 3\left(2y-x^2\right)$$

$$y' = \frac{2y-x^2}{y^2-2x}$$

$$y'\Big|_{\left(\frac{4}{3}, \frac{8}{3}\right)} = \frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)} = \dots$$

$$= \frac{\frac{16}{9}-\frac{16}{3}\cdot\frac{3}{3}}{\frac{3}{3}\cdot\frac{8}{3}-\frac{64}{9}} = \frac{\frac{-32}{9}}{\frac{-40}{9}} = \frac{32}{40} = \boxed{\frac{4}{5}}$$