1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?

$\frac{dV}{dt} = ?$ when $h = 5$ in ; $\frac{dh}{dt} = \frac{-1 \text{ in}}{5 \text{ min}}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{5}\right)^2 h$

$V = \frac{\pi}{3}\left(\frac{h^2}{25}\right)h$

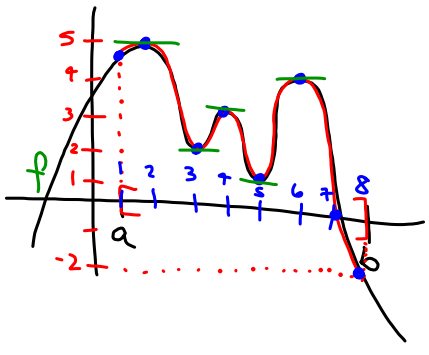
$\frac{r}{h} = \frac{2}{10} \Rightarrow r = \frac{h}{5}$

$\frac{dV}{dt} = \frac{\pi}{75}(3h^2)\frac{dh}{dt} \Leftarrow V = \frac{\pi}{75}h^3$

$= \frac{\pi}{25} \cdot h^2 \cdot \frac{dh}{dt} = \frac{\pi}{25} \cdot 5^2 \cdot \frac{-1}{5} = \boxed{\frac{-\pi}{5} \text{ in}^3/\text{min}}$

3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



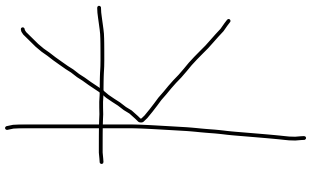
relative minima: (3,2), (5,1)

relative maxima: (2,5), (4,3), (6,4)

absolute maximum: 5 @ (2,5)

absolute minimum: -2 @ (8,-2)

$f(x)$ has a relative maximum or minimum when $f'(x) = 0$. or $f'(x)$ is undefined.



We call such x-values **Critical #'s of f**.

3.1 Find the absolute max & min on the closed interval.

28. $h(t) = \frac{t}{t-2}$, $[3,5]$

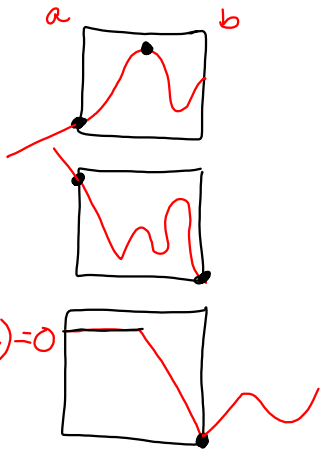
$h'(t) = \frac{(t-2)(1)-t(1)}{(t-2)^2} = \frac{-2}{(t-2)^2}$

critical #'s (t-values such that $h'(t)=0$ or $h'(t)$ is undefined)

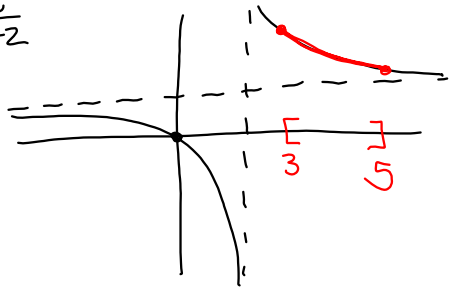
$t = 2 \notin [3,5]$

$h(3) = \frac{3}{3-2} = 3 \leftarrow$ abs max

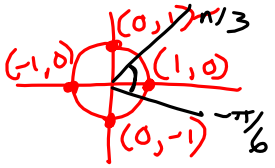
$h(5) = \frac{5}{5-2} = \frac{5}{3} \leftarrow$ abs min



$\frac{t}{t-2}$



30. $g(x) = \sec x$, $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$



Find the absolute max & min on the closed interval.

$g'(x) = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$g'(x) = 0$ when $\sin x = 0$

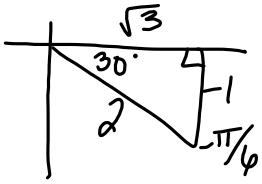
& $g'(x)$ is undefined when $\cos x = 0$

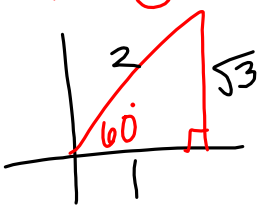
critical #: 0

$\sec(-\pi/6) = 2/\sqrt{3}$

$\sec(0) = \frac{1}{1} = 1 \leftarrow$ abs. min.

$\sec(\pi/3) = 2 \leftarrow$ abs. max.





$1 < 3 < 4$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

$1 < \sqrt{3} < 2$

$1 > \frac{1}{\sqrt{3}} > \frac{1}{2}$

$2 > \frac{2}{\sqrt{3}} > 1$