22. $f(x) = x^3 - 12x$, $[0, 4]$

Find the absolute max & min on the closed interval.

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

critical # in $[0,4]$: 2

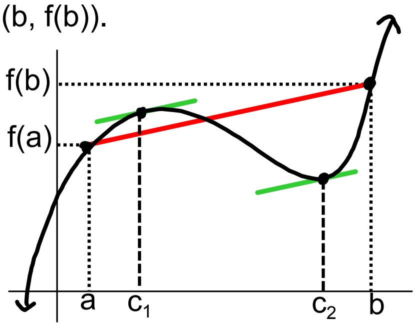
$f(0) = 0$

$f(2) = 8 - 24 = -16 \leftarrow$ abs. min.

$f(4) = 64 - 48 = 16 \leftarrow$ abs. max

3.2 Rolle's Theorem & The Mean Value Theorem

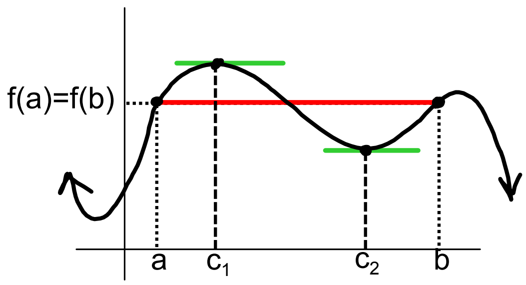
The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



If f is continuous on $[a,b]$, & differentiable on (a,b), there exists at least one $c \in (a,b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

<u>Rolle's Theorem</u> is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



If f is cts. on $[a,b]$ & diff. on (a,b) & $f(a)=f(b)$, then $\exists\ c \in (a,b)$ such that $f'(c)=0$.

If $f(a)=f(b)$, then $\dfrac{f(b)-f(a)}{b-a}=0$

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x)=\frac{x+5}{x-2}\ , \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x)=|x-2|\ , \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4$, $[1,4]$

Is f continuous on $[1,4]$? yes
Is f diff on $(1,4)$? yes
} true for all polynomials

Is $f(1) = f(4)$?
$f(1) = 1 - 5 + 4 = 0$
$f(4) = 16 - 20 + 4 = 0$
} yes

Rolle's Theorem applies
$\Rightarrow$ there exists $c \in (1,4)$ st. $f'(c) = 0$

$f'(x) = 2x - 5$
$2x - 5 = 0$
$2x = 5$
$x = 5/2$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \frac{x+1}{x}$, $\left[\frac{1}{2}, 2\right]$

Steps to solve MVT problems:
1. Is f continuous on [a,b]?
2. Is f differentiable on (a,b)?
} yes $\Rightarrow$ MVT applies $\Rightarrow \exists c \in (\frac{1}{2}, 2)$ st. $f'(c) = \frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}}$
3. Find (f(b)-f(a))/(b-a) $= -1$
4. Find f'(x) $= -1/x^2$
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$$\frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{\frac{4}{2} - \frac{1}{2}} = \frac{\frac{3}{2} - \frac{3}{2} \cdot \frac{2}{1}}{\frac{3}{2}} = 1 - 2 = -1$$

$f(x) = \frac{x+1}{x}$

$f'(x) = \frac{x(1) - (x+1)1}{x^2}$

$= \frac{-1}{x^2}$

$\frac{-1}{x^2} = -1$

$1 = x^2$

$\pm 1 = x$

$x = 1$

$-1 \notin (\frac{1}{2}, 2)$

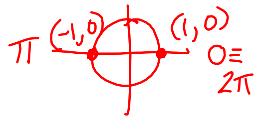
38. $f(x) = 2\sin x + \sin 2x \,,\ [0, \pi]$

Is f cts. on $[0,\pi]$ & diff on $(0,\pi)$? yes

$\Rightarrow$ MVT applies

$$\frac{f(\pi)-f(0)}{\pi - 0} = \frac{(2\sin\pi + \sin 2\pi) - (2\sin 0 + \sin 2(0))}{\pi} = \frac{0}{\pi} = 0$$

$f'(x) = 2\cos x + 2\cos 2x$



$2\cos x + 2\boxed{\cos 2x} = 0$

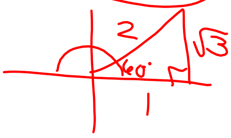
$2\cos x + 2\boxed{(2\cos^2 x - 1)} = 0$

$\cancel{2}[2\cos^2 x + \cos x - 1] = \cancel{0}$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = \pi/3$ $\qquad$ $\cancel{x = \pi \notin (0, \pi)}$



32. $f(x) = x(x^2 - x - 2) \quad [-1, 1]$

f is a polynomial, & hence continuous & differentiable on $(-\infty, \infty)$ $\Rightarrow$ MVT applies.

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{1(1-1-2) - (-1)(1+1-2)}{1+1} = \frac{-2-0}{2} = -1$$

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -1/3$, $\cancel{x} \notin (-1, 1)$

3.1 - Find the absolute extrema on the closed interval

21. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

27. $h(s) = \frac{1}{s-2}$, $[0, 1]$

35. $y = 3\cos x$, $[0, 2\pi]$

3.1 #21,27,35

3.2 #13,15,19; 39,43,45