3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f′* and *f″* tell us about *f*?

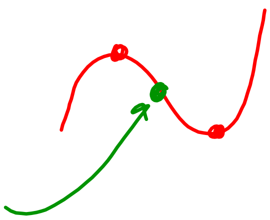
Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.

f'	f
$+$	increasing
$-$	decreasing

f''	f'	f
$+$	increasing	concave up
$-$	decreasing	concave down

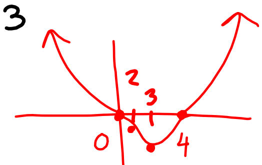
f′(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f′(x) is undefined) are called critical numbers.

f″(x)=0 when f changes concavity.
The points where concavity changes are called inflection points.



3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$



$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

critical #'s: 0, 3

$f'(-1), f'(1), f'(4)$

$- \quad 0 \quad - \quad 3 \quad +$

f has a relative minimum @ $(3, -27)$

f is decreasing on $(-\infty, 3)$

f is increasing on $(3, \infty)$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$f''(-1), f''(1), f''(3)$

$+ \quad 0 \quad - \quad 2 \quad +$

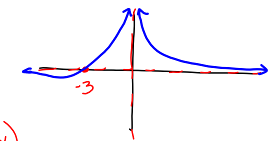
f has inflection points @ $(0, 0)$ & $(2, -16)$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

f is concave down on $(0, 2)$

3.3

30. $f(x) = \dfrac{x+3}{x^2}$



$f'(x) = \dfrac{x^2(1) - (x+3)(2x)}{(x^2)^2}$

$= \dfrac{\cancel{x}(x - 2(x+3))}{x^{\cancel{4}3}}$

$= \dfrac{-x-6}{x^3}$

$\dfrac{-1(x+6)}{x^3} = 0$

critical #'s: $-6, 0$

$f'(-7), f'(-1), f'(1)$

$- \quad -6 \quad + \quad 0 \quad -$

f is decreasing on $(-\infty, -6) \cup (0, \infty)$

f is increasing on $(-6, 0)$

f has a relative minimum @ $(-6, \frac{-1}{12})$

$f''(x) = \dfrac{x^3(-1) - (-x-6)(3x^2)}{(x^3)^2}$

$= \dfrac{-x^2(x + 3(-x-6))}{x^6}$

$= \dfrac{-(-2x-18)}{x^4}$

$= \dfrac{2(x+9)}{x^4}$

$f''(-10), f''(-1), f''(1)$

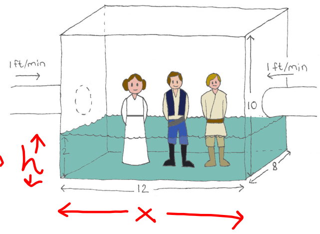
$- \quad -9 \quad + \quad 0 \quad +$

f has an inflection point @ $(-9, \frac{-2}{27})$

f is concave down on $(-\infty, -9)$

f is concave up on $(-9, 0) \cup (0, \infty)$

Leia, Han, and Luke are trapped in a rectangular room 8 feet deep and 10 feet tall. Two opposing walls are closing in at a rate of 1 foot per minute. If the water in the room is 2 feet deep when the moving walls are 12 feet apart, how fast is the water level rising when it reaches the top of Han Solo's head, if Han is 6 feet tall?



$h = 2$ ft when $x = 12$ ft ; $\frac{dx}{dt} = -2$ ft/min

$\frac{dh}{dt} = ?$ when $h = 6$ ft

$V = xh \cdot 8 \Big|_{\substack{x=12 \\ h=2}} = 192 \text{ ft}^3$

$192 = 8xh$

$24 = xh$

when $h = 6$,
$x = \frac{24}{6} = 4$

$24 = xh$

$h = 24x^{-1}$

$$\frac{dh}{dt} = \frac{-24}{x^2} \cdot \frac{dx}{dt} = \frac{-24}{(4)^2} \cdot (-2) = \frac{48}{16} = \boxed{3 \text{ ft/min}}$$