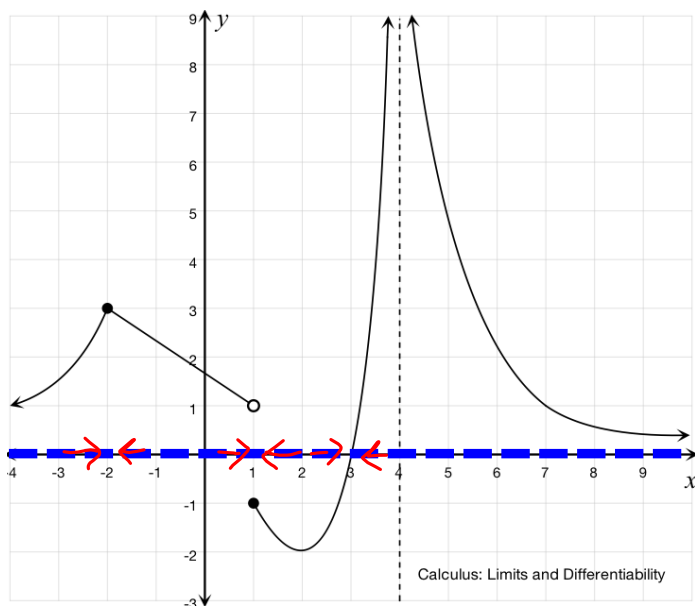


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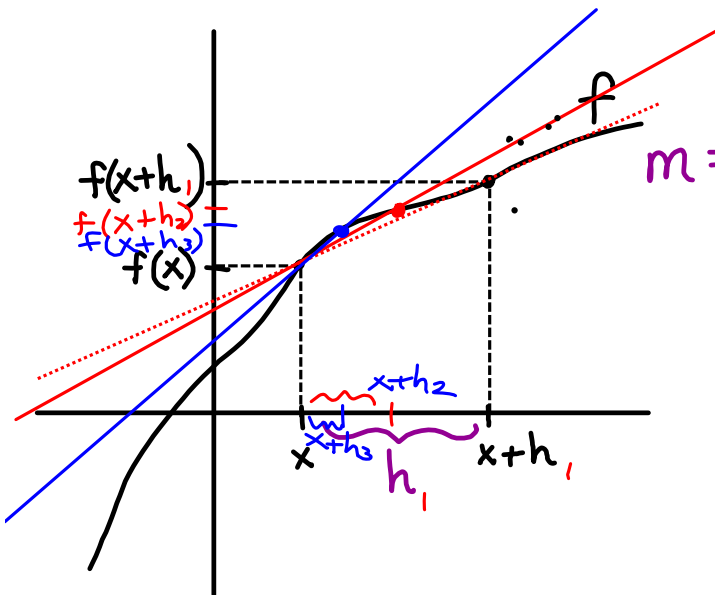
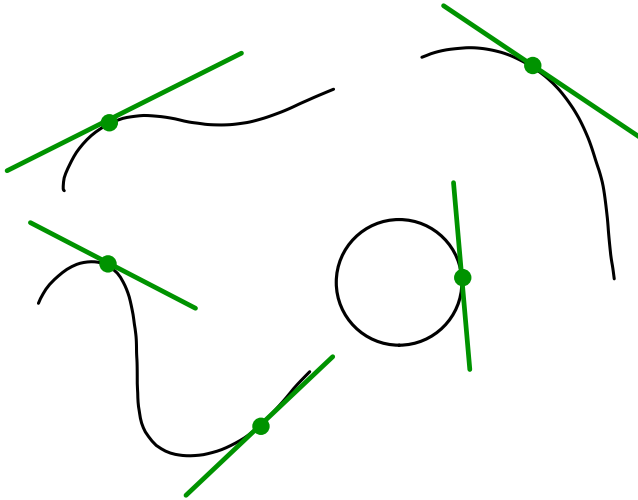
Read sections 1.1 and 1.2 in your textbook

Textbook problems from section 1.2 #1-6 all, 15-22 all 33,34,39,41



| as $x$ approaches..       | $f(x)$ approaches... |
|---------------------------|----------------------|
| -2                        | 3                    |
| $1^-$<br>(from the left)  | 1                    |
| $1^+$<br>(from the right) | -1                   |
| 3                         | 0                    |
| $-\infty$                 | 0                    |
| $\infty$                  | 0                    |
| 4                         | $\infty$             |

tangent lines

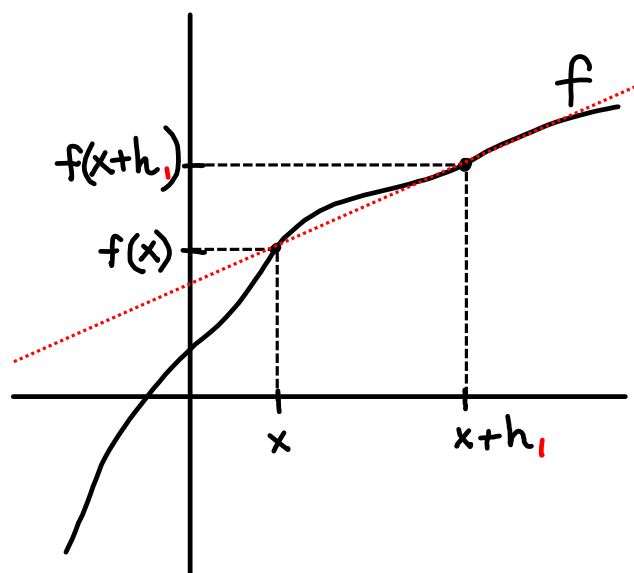


secant line

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

the difference quotient



tangent line

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

is the slope of the tangent line to  $f$  at  $(x, f(x))$ .

$\Delta x$  "delta x"  
means change in  $x$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

↑ treated as a single variable

1.2

$$f(x) = \frac{x-2}{x^2-4}, \quad x \neq 2, -2$$

$$\lim_{x \rightarrow 2} f(x) = \frac{1}{4}$$

What happens to  $f(x)$  as  $x$  approaches 2?

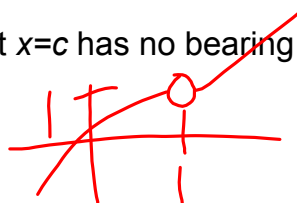
|        |        |        |        |      |        |        |        |
|--------|--------|--------|--------|------|--------|--------|--------|
| $x$    | 1.9    | 1.99   | 1.999  | 2    | 2.001  | 2.01   | 2.1    |
| $f(x)$ | 0.2564 | 0.2506 | 0.2501 | 0.25 | 0.2499 | 0.2494 | 0.2439 |

### Informal Description of the Limit

If  $f(x)$  becomes arbitrarily close to a single number  $L$  as  $x$  approaches  $c$  from either side, the **limit** of  $f(x)$ , as  $x$  approaches  $c$ , is  $L$ .

$$\lim_{x \rightarrow c} f(x) = L$$

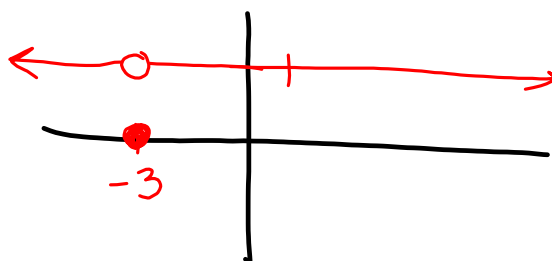
Note: the existence or nonexistence of  $f(x)$  at  $x=c$  has no bearing on the existence of the limit as  $x$  approaches  $c$ .



A function can be undefined for a certain value of  $c$  with the limit as  $x$  approaches  $c$  still defined.

$$\lim_{x \rightarrow -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

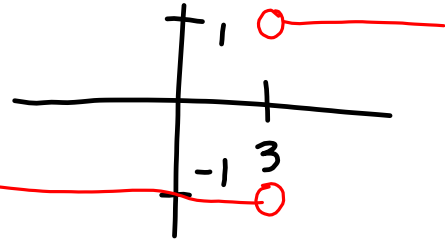
$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$



$$\lim_{x \rightarrow -3} f(x) = \boxed{1}$$

$$\lim_{x \rightarrow 3} \frac{|x-3|}{x-3} = \text{undefined} \quad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

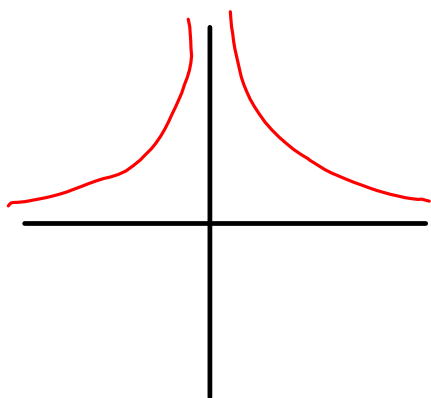
$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3 > 0 \\ & x > 3 \\ -\frac{(x-3)}{x-3} = -1, & x-3 < 0 \\ & x < 3 \end{cases}$$



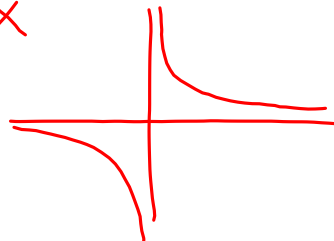
$$\lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^-} -\frac{(x-3)}{x-3} = -1$$

$$\lim_{x \rightarrow 3^+} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^+} \frac{x-3}{x-3} = 1$$

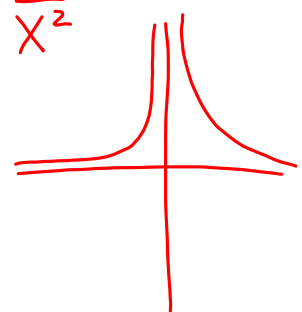
$$\lim_{x \rightarrow 0} \frac{1}{x^4} = \infty$$



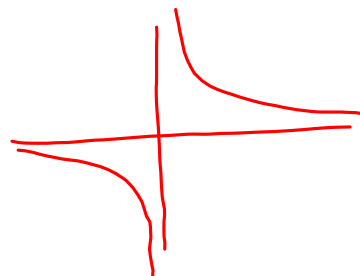
$$\frac{1}{x}$$



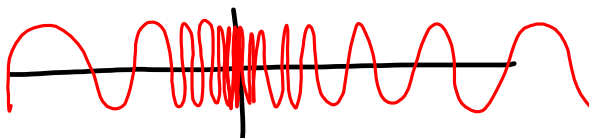
$$\frac{1}{x^2}$$



$$\frac{1}{x^3}$$



$\lim_{x \rightarrow 0} \sin \frac{1}{x}$

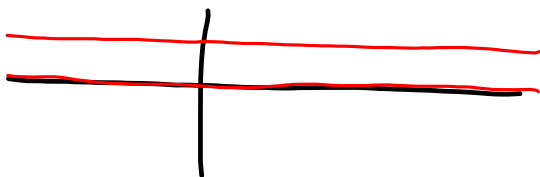


|                    |                          |                            |                           |                  |                  |                   |
|--------------------|--------------------------|----------------------------|---------------------------|------------------|------------------|-------------------|
| $x$                | $\frac{2}{\pi}$          | $\frac{2}{3\pi}$           | $\frac{2}{5\pi}$          | $\frac{2}{7\pi}$ | $\frac{2}{9\pi}$ | $\frac{2}{11\pi}$ |
| $\sin \frac{1}{x}$ | $\sin \frac{\pi}{2} = 1$ | $\sin \frac{3\pi}{2} = -1$ | $\sin \frac{5\pi}{2} = 1$ | $-1$             | $1$              | $-1$              |

undefined

### "Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$



limits are undefined for all x-values