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HW due Tues (8th per)/Wed (7th per): 1.2 #1-6 all, 15-22 all, 33,34,39,41

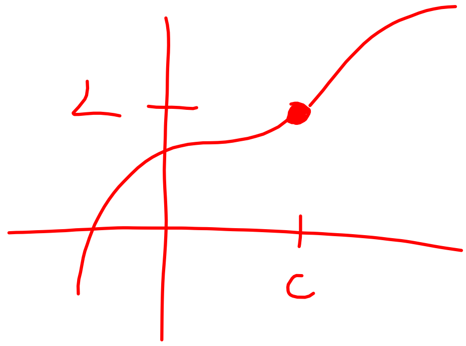
Due Fri: 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 evaluating limits analytically

- 1.3 #63-73 odd; 89, 90 limits with trig, squeeze theorem
- 1.4 #1-19 odd; limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 misc. continuity problems

1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is continuous at c.



Evaluating Limits Analytically

Basic Limits

$Let\ \ b,c \in \mathbb{R}, \quad n > 0\ an\ integer, \quad f,g - functions, \quad \lim_{x\to c} f(x) = L, \quad \lim_{x\to c} g(x) = K$

1. Constant $\quad \lim_{x\to c} b = b$
2. Identity $\quad \lim_{x\to c} x = c$
3. Polynomial $\quad \lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\quad \lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\quad \lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\quad \lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\quad \lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K} \quad , \quad K \neq 0$
8. Power $\quad \lim_{x\to c}[f(x)]^n = L^n$

$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

$\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} \quad , \lim_{x\to c} g(x) \neq 0$

$\lim_{x\to c} [f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$\lim_{x\to c} a = a$

$\lim_{x\to c} x = c$

$\lim_{x\to c} x^n = c^n$

$\lim_{x\to 5} (-3) = -3$

$\lim_{x\to -\pi} x = -\pi$

$\lim_{x\to -1} x^5 = (-1)^5 = -1$

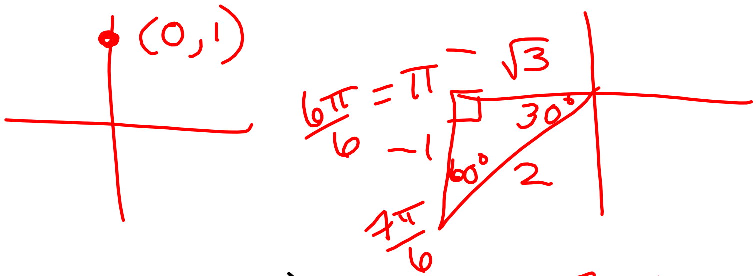
1.3

12. $\lim_{x\to 1}\left(3x^3-2x^2+4\right) = 3(1)^3-2(1)^2+4$

$= 3-2+4 = \boxed{5}$

18. $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \frac{2}{-1} = \boxed{-2}$

30. $\lim_{x\to 1}\sin\frac{\pi x}{2} = \sin\frac{\pi}{2} = \boxed{1}$



36. $\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$

38. $\lim\limits_{x\to c} f(x) = \frac{3}{2}$; $\lim\limits_{x\to c} g(x) = \frac{1}{2}$

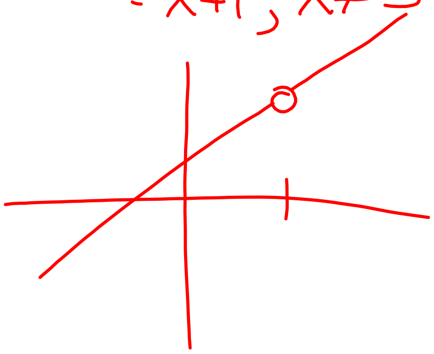
(a) $\lim\limits_{x\to c}\left[4f(x)\right] = 4\cdot\frac{3}{2} = \boxed{6}$

(b) $\lim\limits_{x\to c}\left[f(x)+g(x)\right] = \frac{3}{2}+\frac{1}{2} = \frac{4}{2} = \boxed{2}$

(c) $\lim\limits_{x\to c}\left[f(x)g(x)\right] = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{3}{4}}$

(d) $\lim\limits_{x\to c}\frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2}\cdot\frac{2}{1} = \boxed{3}$

$$\lim_{x\to 3}\frac{x^2-2x-3}{\underbrace{x-3}_{= x+1,\ x\neq 3}} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$$



$$= \lim_{x\to 3}(x+1)$$

$$= \boxed{4}$$

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$a+\sqrt{b} \to a-\sqrt{b}$

$\sqrt{a}+b \to \sqrt{a}-b$

$(a-b)(a+b) = a^2-b^2$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x\to 4} (\sqrt{x}+2) = \sqrt{4}+2 = 2+2 = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2+3(x+h)+1-(2x^2+3x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+3x+3h+1-2x^2-3x-1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h+\cancel{1}-\cancel{2x^2}-\cancel{3x}-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4x+2h+3 = \boxed{4x+3}$$

$f(x) = x^3$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} =$$

1
1 1
1 2 1
1 3 3 1

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$

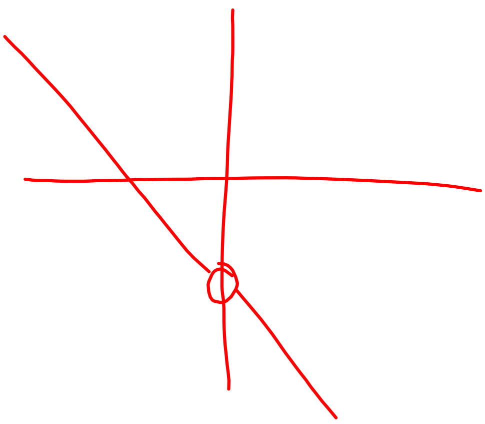
$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}} = \boxed{3x^2}$$

1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2-3x}{x} = \dfrac{x(x-3)}{x} = x-3, \ x \neq 0$

(a) $\lim\limits_{x\to -2} h(x) = -2-3 = \boxed{-5}$

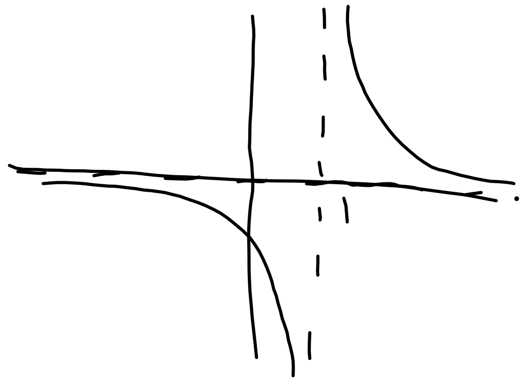
(b) $\lim\limits_{x\to 0} h(x) = 0-3 = \boxed{-3}$



44. $\lim\limits_{x\to 1} \dfrac{x}{x^2-x} = \lim\limits_{x\to 1} \dfrac{\cancel{x}}{\cancel{x}(x-1)}$

$= \lim\limits_{x\to 1} \dfrac{1}{x-1}$

$= \boxed{\text{does not exist}}$



48. $\lim\limits_{x\to -1} \dfrac{x^3+1}{x+1} = \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= (-1)^2-(-1)+1 = \boxed{3}$

54. $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$$= \lim_{x\to 0} \frac{\cancel{2}+\cancel{x}-\cancel{2}}{\cancel{x}\left(\sqrt{2+x}+\sqrt{2}\right)} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$$