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HW due Tues (8th per)/Wed (7th per): 1.2 #1-6 all, 15-22 all, 33,34,39,41

Due Fri 3/10: 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 evaluating limits analytically

Due Tues (8th per)/Wed (7th per):

- 1.3 #63-73 odd; 89, 90 — limits with trig, squeeze theorem
- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems

Quiz - early next week

Test - late next week or early the following - Ch 1 Limits

The Squeeze Theorem:

If $f(x) \leq g(x) \leq h(x)$ *and* $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$,

Then $\lim_{x \to c} g(x) = L$.

Special Limits Derived by Squeeze Theorem:

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1} \; ; \; \boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$$

Memorize!!

78. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\lim_{ax\to 0} \frac{\sin ax}{ax} = 1$$

$$x\to 0 \Rightarrow ax\to 0$$

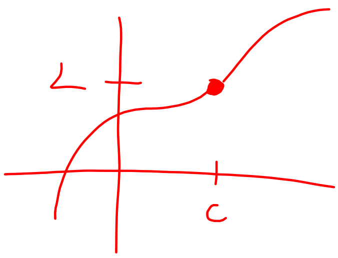
$$= \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}\cdot 2\cancel{x}}{\frac{\sin 3x}{3x}\cdot 3\cancel{x}}$$

$$= \frac{\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)}{\left(\lim_{x\to 0}\frac{\sin 3x}{3x}\right)}\cdot\left(\lim_{x\to 0}\frac{2}{3}\right)$$

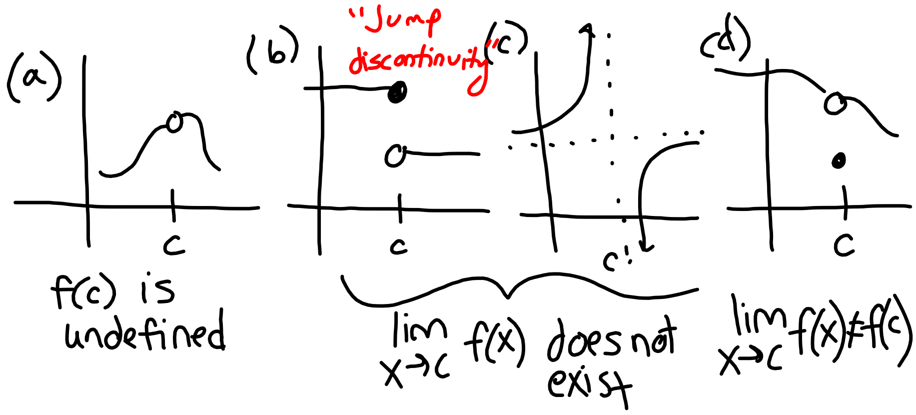
$$= \frac{1}{1}\cdot\frac{2}{3} = \boxed{\frac{2}{3}}$$

1.3 Evaluating Limits Analytically

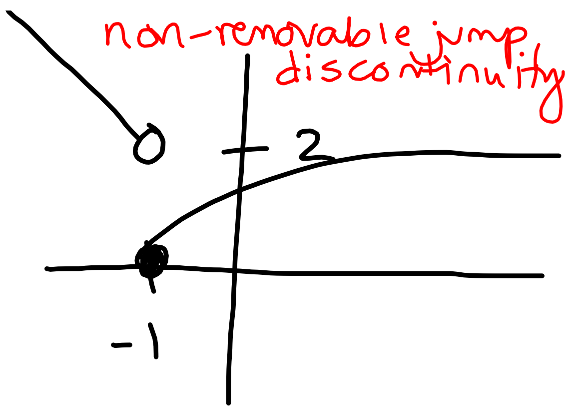
If $\lim_{x\to c} f(x) = f(c)$,
we say that $f(x)$ is
continuous at c.



1.4 Continuity and One-Sided Limits



These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x\to -1^-} f(x) = 2$$

$$\lim_{x\to -1^+} f(x) = 0$$

$$\lim_{x\to -1} f(x) =$$ undefined/ does not exist

One-Sided Limits

$\lim_{x\to c^+} f(x) = L$ *limit from the right*

$\lim_{x\to c^-} f(x) = L$ *limit from the left*

$$\boxed{\lim_{x\to c} f(x) = L \text{ if and only if } \lim_{x\to c^-} f(x) = L = \lim_{x\to c^+} f(x)}$$

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x\to c} f(x) = f(c)$$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

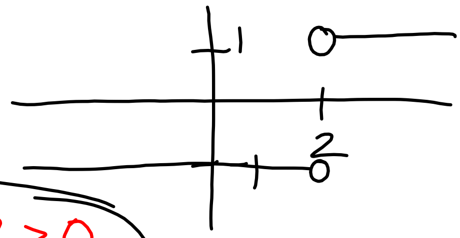
10. $$\lim_{x\to 4^-} \frac{(\sqrt{x}-2)}{(x-4)} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$$

$$= \lim_{x\to 4^-} \frac{\cancel{x-4}\ 1}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4^-} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

f has a removable discontinuity @ 4

12. $\displaystyle\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$



$$\frac{|x-2|}{x-2} = \begin{cases} \dfrac{x-2}{x-2} = 1, & x-2>0 \\ & x>2 \\ \dfrac{-(x-2)}{x-2} = -1, & x-2<0 \\ & x<2 \end{cases}$$

1.4

Discuss the [dis]continuity of the function.

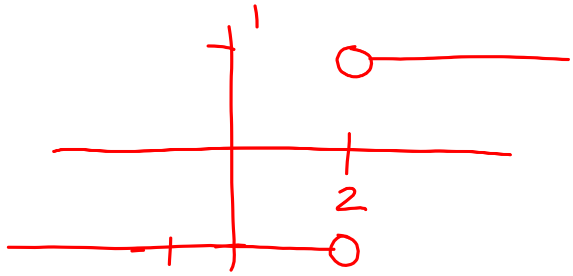
$$f(x) = \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

removable discontinuity @ $x=2$ (hole)

non-removable discontinuity @ $x=-1$ (vertical asymptote)

f is continuous on: $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

$$f(x) = \frac{|x-2|}{x-2}$$

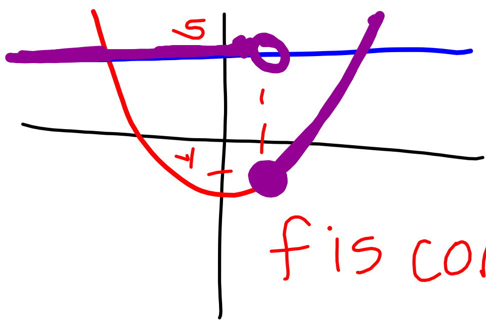


f has a non-removable jump discontinuity @ $x=2$

f is continuous on $(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

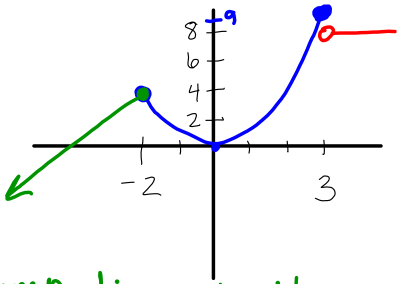
$1^2 - 2 = -1$



f has a non-removable jump discontinuity @ $x = 1$

f is continuous on $(-\infty, 1) \cup [1, \infty)$

$$f(x) = \begin{cases} \overset{-2+6=4}{x+6}, & x \le -2 \\ x^2 \quad (-2)^2 = 4, \; 3^2 = 9 & -2 < x \le 3 \\ 8 & x > 3 \end{cases}$$



f has a non-removable jump discontinuity @ $x=3$

f is continuous on $(-\infty, 3] \cup (3, \infty)$