Diff Cal - 1.4-1.5 - Intermediate Value Theorem & Infinite Limits — March 14, 2017

Sign up for Khan Academy with coach code 3XDPSR.

HW due Tues (8th per)/Wed (7th per): 1.2 #1-6 all, 15-22 all, 33,34,39,41

Due Fri 3/10: 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 evaluating limits analytically

Due Tues (8th per)/Wed (7th per):
1.3 #63-73odd,89,90; 1.4 #1-27odd,28-30all, 43-48all, 57,61,65,69,99,102

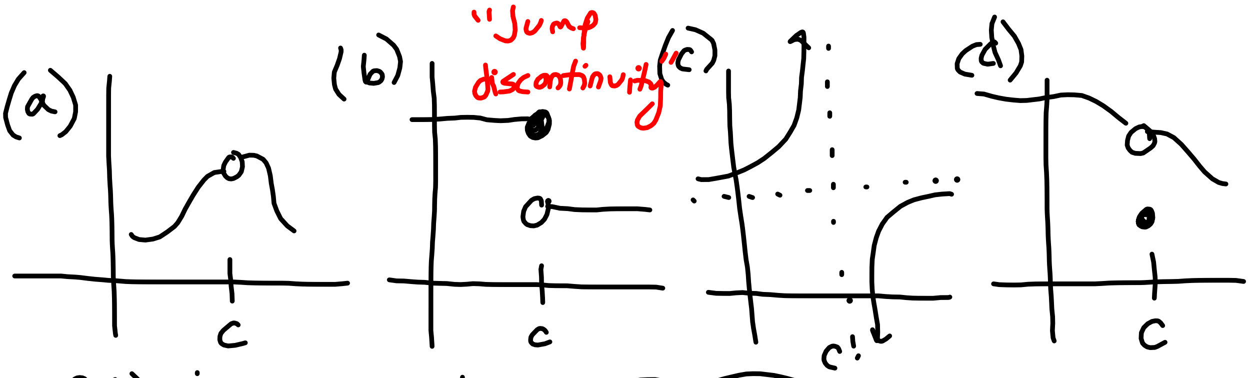
Breakdown by problem type:

- 1.3 #63-73 odd; 89, 90 — limits with trig, squeeze theorem
- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems

Test - Ch 1 Limits - Friday

1.5# 1,3,23,29-57 odd due Fri?

1.4 Continuity and One-Sided Limits



f(c) is undefined

$\lim_{x\to c} f(x)$ does not exist

$\lim_{x\to c} f(x) \neq f(c)$

These are all <u>discontinuities</u>
(a) and (d) are <u>removable</u>
(b) and (c) are <u>nonremovable</u>

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x\to c} f(x) = f(c)$$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is everywhere continuous.

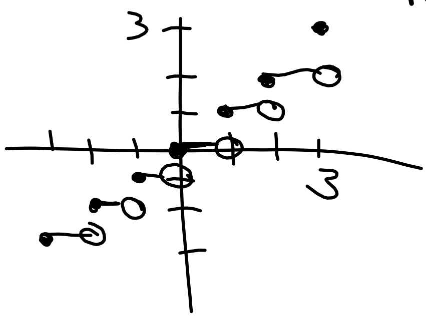
Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

$$f(x) = \begin{cases} \dfrac{|x-3|}{3-x}, & |x-3| > 5 \\ x^2 - 3, & -2 \le x \le 8 \end{cases}$$

The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x



22. $\lim_{x\to2^+} 2x - [\![x]\!]$

$= \lim_{x\to2^+} 2x - \lim_{x\to2^+} [\![x]\!]$

$= 4 - 2 = \boxed{2}$

24. $\lim_{x \to 1} \left(1 - [\![-\frac{x}{2}]\!]\right)$

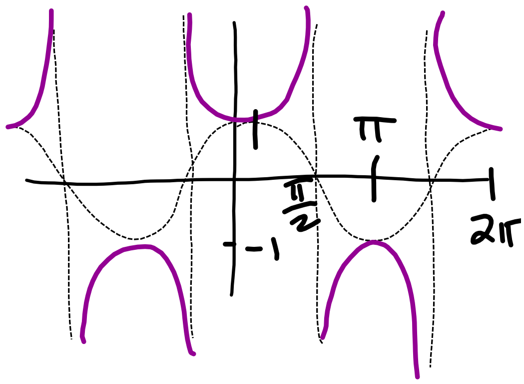
$= \lim_{x \to 1} (1) - \lim_{x \to 1} [\![-\frac{x}{2}]\!]$

$= 1 - (-1) = \boxed{2}$

20. $\lim_{x \to \frac{\pi}{2}} \sec x$ does not exist

$= \lim_{x \to \frac{\pi}{2}} \frac{1}{\cos x}$

$= \frac{1}{\lim_{x \to \frac{\pi}{2}} \cos x} = \frac{1}{0}$

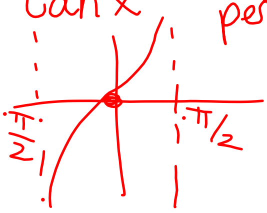


$\lim_{x \to \frac{\pi}{2}^-} \sec x = \infty$

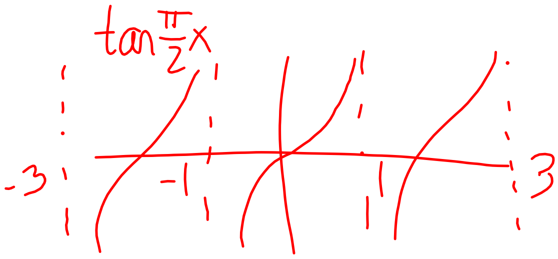
$\lim_{x \to \pi/2^+} \sec x = -\infty$

52. $f(x)=\tan\frac{\pi x}{2}$ period $=\frac{\pi}{\pi/2}=2$

discuss the (dis)continuity



period $=\pi$



$\tan\frac{\pi}{2}x$ has non-removable vertical asymptotes at $x=2k-1$, $k\in\mathbb{Z}$

$\tan\frac{\pi}{2}x$ is continuous on all intervals of the form $(2k-1, 2k+1)$

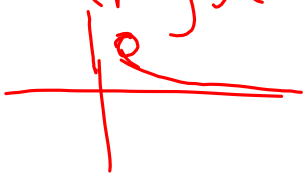
$(2k+1, 2k+3)$

62. $f(x)=\frac{1}{\sqrt{x}}$, $g(x)=x-1$

Discuss the continuity of $f(g(x))$.

$(f\circ g)(x)$

$f(g(x))=\frac{1}{\sqrt{x-1}}$



domain is $(1,\infty)$

$f(g(x))$ is continuous on its domain $(1,\infty)$

64. $f(x) = \sin x$; $g(x) = x^2$

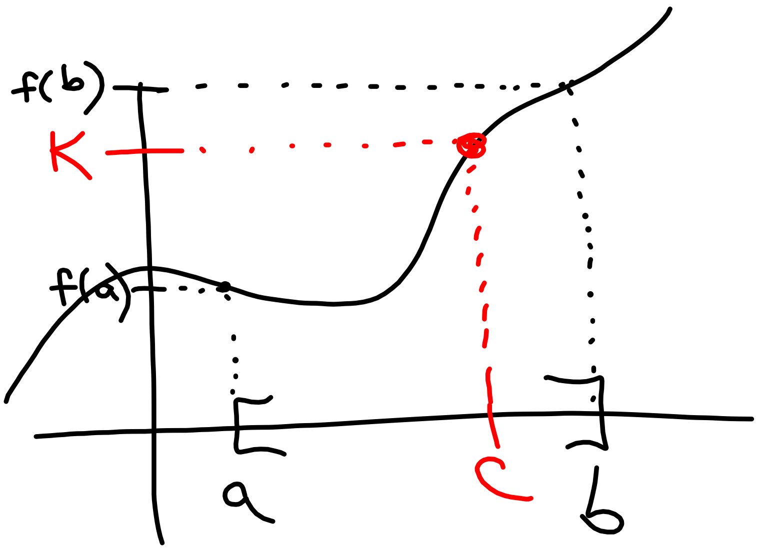
discuss the continuity of $f(g(x))$

$f(g(x)) = \sin(x^2)$

continuous on $(-\infty, \infty)$

<u>Intermediate Value Theorem</u>

If *f* is continuous on the closed interval [*a*,*b*] and *k* is any number between *f(a)* and *f(b)*, then there is at least one number *c* in [*a*,*b*] such that *f(c)=k*.

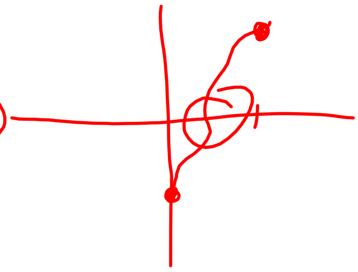


Does the IVT guarantee a zero in the given interval?

76. $f(x) = x^3 + 3x - 2 \quad , [0, 1]$

$f(0) = -2 < 0$

$f(1) = 1 + 3 - 2 = 2 > 0$



$\Rightarrow$ IVT guarantees a zero on $[0, 1]$

84. $f(x) = x^2 - 6x + 8 \; ; [0, 3] \; f(c) = 0$

86. $f(x) = \frac{x^2+x}{x-1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$

$f\left(\frac{5}{2}\right) = \frac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \frac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \frac{35}{\cancel{4}_2} \cdot \frac{\cancel{2}}{3} = \frac{35}{6}$

< 6

$f(4) = \frac{4^2 + 4}{4 - 1} = \frac{20}{3} > 6$

IVT guarantees a c such that $f(c) = 6$

$\frac{x^2+x}{x-1} = 6$

$x^2 + x = 6(x-1)$

$x^2 + x = 6x - 6$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2)$

$x = \cancel{2}, 3$

not in $[5/2, 4]$

1.5

Infinite Limits

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty$ *and* $\lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

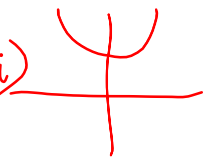
2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$ $(x-2i)(x+2i)$

N/A



24. $h(x) = \dfrac{x^2-4}{\underbrace{x^3+2x^2}_{x^2(x+2)}+\underbrace{x+2}_{1(x+2)}} = \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x^2+1)}$ hole @ -2

N/A

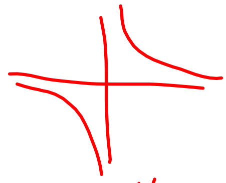
28. $g(\theta) = \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\theta} = \dfrac{\sin\theta}{\theta}\cdot\dfrac{1}{\cos\theta}$

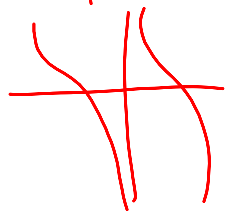
V.A's @ all odd multiples of $\pi/2$

42. $\lim_{x\to0^-}\left(x^2-\frac{1}{x}\right) = \lim_{x\to0^-} x^2 - \lim_{x\to0^-}\frac{1}{x}$

$= 0 - (-\infty)$

$= \boxed{\infty}$





46. $\lim_{x\to0}\frac{x+2}{\cot x} = \frac{\lim_{x\to0}(x+2)}{\lim_{x\to0}\cot x} = \frac{2}{\pm\infty} = \boxed{0}$

$= \lim_{x\to0}(x+2)\cdot\lim_{x\to0}\tan x$

$2\cdot 0 = \boxed{0}$