Due Wed. 3/22:

2.1 #1-41 odd;

Due Mon. 3/27:

2.1 #65-89 odd
2.2 #3-67 odd;

Due Wed for 8th per, Thurs for 7th per:

2.2 #87-95 odd; 97-100 all; 105,106,111,113,115

2.1 The Derivative & The Tangent Line Problem

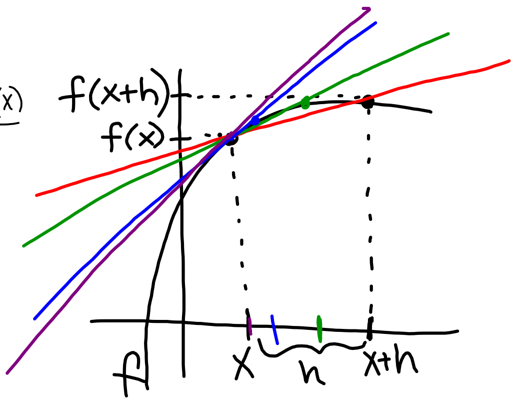
secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$f'(x)$	"f prime of x"
$\frac{dy}{dx}$	"derivative of y with respect to x"
y'	"y prime"
$\frac{d}{dx}[f(x)]$	"the derivative with respect to x of $f(x)$"
$D_x[y]$	"the partial derivative with respect to x of y"

2.1 Differentiability & Continuity

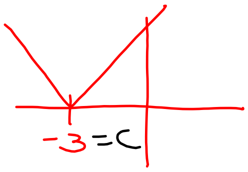
Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$

$f(x) = |x + 3|$



is f differentiable @ $x=-3$ & why?

$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$

$\lim_{x \to c^-} \frac{f(x)-f(c)}{x-c}$ slope of tangent line from the left

$\lim_{x \to c^+} \frac{f(x)-f(c)}{x-c}$ slope of tangent from the right

$$\frac{f(x)-f(c)}{x-c} = \frac{|x+3|-|-3+3|}{x-(-3)} = \frac{|x+3|}{x+3} = \begin{cases} \frac{x+3}{x+3} = 1, & x > -3 \\ \frac{-(x+3)}{x+3} = -1, & x < -3 \end{cases}$$

$\lim_{x \to -3^-} = -1$

$\lim_{x \to -3^+} = +1$

Since left- & right-hand limits are different, the limit in general does not exist and the derivative defined by that limit does not exist

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,
for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

$\sqrt[n]{x} = x^{1/n}$ $x^0 = x$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{1}{x^n} = x^{-n}$

$\frac{d}{dx}[x] = 1$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[cx] = c$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \sin h - \sin x(1-\cos h)}{h}$$

$$= \lim_{h\to 0} \left[\cos x \cdot \frac{\sin h}{h} - \sin x \cdot \frac{1-\cos h}{h}\right]$$

$$= \cos x \cdot 1 - \sin x \cdot 0$$

$$= \boxed{\cos x}$$

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2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \boxed{\frac{-15}{8}x^{-4} - 2\sin x}$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x)=\dfrac{2}{\sqrt[3]{x}}+3\cos x = 2x^{-1/3}+3\cos x$

$f'(x)=\boxed{-\frac{2}{3}x^{-4/3}-3\sin x}$

2.2 cont.

$s(t)$ = position

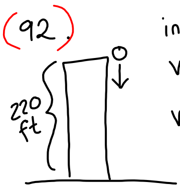
$v(t)=s'(t)$ = velocity

$a(t)=v'(t)=s''(t)$ = acceleration

average velocity: $\dfrac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

(92)



initial velocity $V_0 = -22$ ft/s

$v(3) = ?$

$v(t) = ?$ after falling 108 ft

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$g = -9.8$ m/s²

$= -32$ ft/s²

a = acceleration

V_0 = initial velocity = $v(0)$

S_0 = initial position = $S(0)$

$V_0 = -22$ ft/s , $S_0 = 220$ ft , $a = -32$ ft/s²

$\boxed{S(t) = -16t^2 - 22t + 220}$

$S(t) = 220 - 108 = 112$

$V(t) = S'(t) = -32t - 22$

$a(t) = V'(t) = S''(t) = -32$

$V(t) = -32t - 22$

$V(3) = -32(3) - 22 = -96 - 22 = \boxed{-118 \text{ ft/s}}$

$112 = -16t^2 - 22t + 220$

$16t^2 + 22t - 108 = 0 \quad \Rightarrow t = 2$

$V(2) = -32(2) - 22 = -64 - 22 = \boxed{-86 \text{ ft/s}}$

The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$

Find the rate of change of volume with respect to radius when the radius is 2 cm.

$\frac{dV}{dr} = V'(r) = 4\pi r^2$ = surface area of a sphere

$V'(2) = 4\pi(2)^2 = \boxed{16\pi \text{ cm}^2}$