Due Wed. 3/22: 2.1 #1-41 odd;

Due Mon. 3/27: 2.1 #65-89 odd; 2.2 #3-67 odd;

Due Wed for 8th per, Thurs for 7th per:

2.2 #87-95 odd; 97-100 all; 105,106,111,113,115

Due Monday:

2.3 #1-53odd,63-85odd,91-105odd,111-115odd

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.2 cont.

$s(t)$ = position

$v(t) = s'(t)$ = velocity

$a(t) = v'(t) = s''(t)$ = acceleration

average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}$ d/dx [c]=0

Constant Multiple Rule:

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

Sum & Difference:

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule:

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule:

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

Chain Rule:

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Trig Functions:

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.3 Product & Quotient Rules

$$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\boxed{(fg)' = f'g + fg'}$$

$$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"low dee high less high dee low,
draw the line and square below"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

2.3

6. $g(x) = \sqrt{x}\sin x = (x^{1/2})(\sin x)$

$$g'(x) = (x^{1/2})'(\sin x) + (x^{1/2})(\sin x)'$$

$$= \boxed{\frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x}$$

12. $f(t)=\frac{\cos t}{t^3}=(\cos t)(t^{-3})$

$f'(x)=(-\sin t)(t^{-3})+(\cos t)(-3t^{-4})$

$f'(t)=\frac{t^3(\cos t)'-(\cos t)(t^3)'}{(t^3)^2}$

$=\frac{-t^3\sin t-3t^2\cos t}{t^6}$

$=\frac{t^2(-t\sin t-3\cos t)}{t^6}$

$=\boxed{\frac{-t\sin t-3\cos t}{t^4}}$

$\boxed{(a^m)^n=a^{mn}}$

$(-\sin t)(t^{-3})+(\cos t)(-3t^{-4})$

$=\frac{-\sin t}{t^3}\cdot\frac{t}{t}-\frac{3\cos t}{t^4}=\frac{-t\sin t-3\cos t}{t^4}$

26. $f(x)=\frac{x^3+3x+2}{x^2-1}$

Note: as a product,
$f(x)=(x^3+3x+2)(x^2-1)^{-1}$
we don't know how to differentiate this yet
so we have to use the quotient rule!

$\left[\frac{f}{g}\right]'=\frac{gf'-fg'}{g^2}$

"low dee high, less high dee low
draw the line and square below"

$f'(x)=\frac{(x^2-1)(x^3+3x+2)'-(x^3+3x+2)(x^2-1)'}{(x^2-1)^2}$

$=\boxed{\frac{(x^2-1)(3x^2+3)-(x^3+3x+2)(2x)}{(x^2-1)^2}}$

$=\frac{3x^4+3x^2-3x^2-3-2x^4-6x^2-4x}{(x^2-1)^2}$

$=\boxed{\frac{x^4-6x^2-4x-3}{(x^2-1)^2}}$