Test Friday, 4/28 - derivatives, second derivatives

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation due Fri
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

What happens if...

$$x^2y + y^2x = -2$$

how to find y'?

2.5 Implicit Differentiation

★ $y = f(x)$

y is a function of x

$\frac{d}{dx}[x] = 1$; $\frac{d}{dx}[y] = y'$

6. $x^2y + y^2x = -2$

$\frac{d}{dx}[x^2y + y^2x] = \frac{d}{dx}[-2]$

$\frac{d}{dx}[x^2y] + \frac{d}{dx}[y^2x] = 0$

$\frac{d}{dx}[x^2] \cdot y + x^2 \cdot \frac{d}{dx}[y] + \frac{d}{dx}[y^2] \cdot x + y^2 \cdot \frac{d}{dx}[x] = 0$

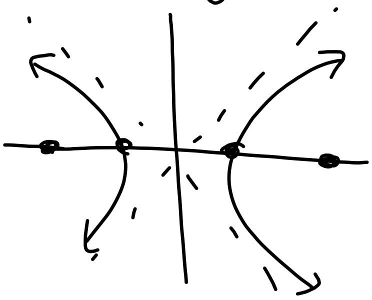
$2xy + x^2y' + 2y \cdot y' \cdot x + y^2 \cdot 1 = 0$

$x^2y' + 2xyy' = -y^2 - 2xy$

$y'(x^2 + 2xy) = -y^2 - 2xy$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16$



$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2y \cdot y' = 0$$

$$\frac{2x}{2y} = \frac{\cancel{2y}y'}{\cancel{2y}}$$

$$\boxed{y' = \frac{x}{y}}$$

8. $\sqrt{xy} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 2y\right]$$

$$\frac{1}{2}(xy)^{-1/2}\left[1 \cdot y + x \cdot y'\right] = 1 - 2y'$$

$$\frac{y}{2\sqrt{xy}} + \frac{xy'}{2\sqrt{xy}} = 1 - 2y'$$

$$\frac{xy'}{2\sqrt{xy}} + 2y' = 1 - \frac{y}{2\sqrt{xy}}$$

$$y'\left(\frac{x}{2\sqrt{xy}} + 2\right) = 1 - \frac{y}{2\sqrt{xy}}$$

$$\boxed{y' = \frac{1 - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2}}$$

10. $2\sin x\cos y = 1$

$\frac{d}{dx}\left[(2\sin x)(\cos y)\right] = \frac{d}{dx}[1]$

$(2\cos x)(\cos y) + (2\sin x)(-\sin y)y' = 0$

$\cancel{2}\cos x\cos y = \cancel{2}y'\sin x\sin y$

$\frac{\cos x\cos y}{\sin x\sin y} = y'$

$\boxed{\cot x\cot y = y'}$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$\cancel{2}(\sin\pi x + \cos\pi y)\cdot(\pi\cos\pi x - \pi y'\sin\pi y) = 0$

$\pi\sin\pi x\cos\pi x - \pi y'\sin\pi x\sin\pi y + \pi\cos\pi x\cos\pi y - \pi y'\sin\pi y\cos\pi y = 0$

$\pi\sin\pi x\cos\pi x + \pi\cos\pi x\cos\pi y = \pi y'\sin\pi x\sin\pi y + \pi y'\sin\pi y\cos\pi y$

$\frac{\cancel{\pi}\cos\pi x\cancel{(\sin\pi x + \cos\pi y)}}{\cancel{\pi}\sin\pi y\cancel{(\sin\pi x + \cos\pi y)}} = y'$

$y' = \boxed{\frac{\cos\pi x}{\sin\pi y}}$

16. $x = \sec\frac{1}{y}$

$$\frac{d}{dx}\left[x\right] = \frac{d}{dx}\left[\sec(y^{-1})\right]$$

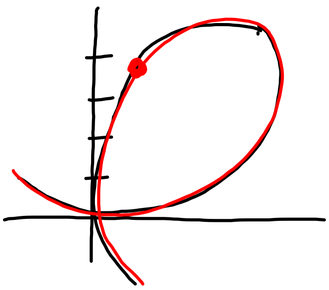
$$1 = \sec(y^{-1})\tan(y^{-1}) \cdot -y^{-2} \cdot y'$$

$$y' = \frac{-y^2}{\sec\frac{1}{y}\tan\frac{1}{y}}$$

32. Folium of Descartes

$x^3 + y^3 - 6xy = 0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}\left[6xy\right]$$

$$3x^2 + 3y^2 \cdot y' = 6y + 6xy'$$

$$3y^2y' - 6xy' = 6y - 3x^2$$

$$y' = \frac{\cancel{3}(2y - x^2)}{\cancel{3}(y^2 - 2x)}\Bigg|_{\left(\frac{4}{3}, \frac{8}{3}\right)} =$$

$$= \frac{2\left(\frac{8}{3}\right) - \left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2 - 2\left(\frac{4}{3}\right)} = \frac{\frac{16}{3} - \frac{16}{9}}{\frac{64}{9} - 8/3} = \frac{48-16}{\cancel{9}} \cdot \frac{\cancel{9}}{64-24} = \frac{32}{40} = \boxed{\frac{4}{5}}$$