Applications of Derivatives

Due Wednesday, 5/3:
2.6 # 11-17 odd - Related Rates

Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)

Next week:
3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

2.6 Related Rates

18. $V = \frac{4}{3}\pi r^3$ Volume of a sphere

$\frac{dr}{dt} = 2 \text{ in/min}$

$\frac{dV}{dt} = ?$ when $r = 6$ in

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2 \cdot \frac{dr}{dt}\right) = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} \cdot 6^2 \cdot 2$$

$$= 288\pi \text{ in}^3/\text{min}$$

22. $V=\frac{1}{3}\pi r^2 h$

$\frac{dr}{dt}=2$ in/min

$h=3r$

$\frac{dV}{dt}=?$ when $r=6$in

volume of a cone

$V=\frac{1}{\cancel{3}}\pi r^2(\cancel{3}r)$

$\cancel{\frac{d}{dt}[V]=\frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]}$

$\cancel{\frac{dV}{dt}=\left(\frac{1}{3}\pi r^2\right)\cdot\frac{dh}{dt}+\frac{1}{3}\pi h\cdot 2r\cdot\frac{dr}{dt}}$

$V=\pi r^3$

$\frac{d}{dt}[V]=\frac{d}{dt}\left[\pi r^3\right]$

$\frac{dV}{dt}=3\pi r^2\cdot\frac{dr}{dt}$

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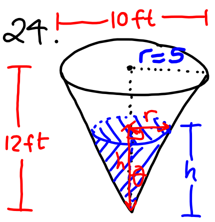
$\frac{\cancel{6}}{6}$

$=3\pi(6)^2\cdot 2$

$=\boxed{216\pi \text{ in}^3/\text{min}}$

24.



$\frac{dV}{dt}=10$ ft³/min

$\frac{dh}{dt}=?$ when $h=8$ ft

$\frac{r}{h}=\frac{5}{12}$ $12r=5h$ $r=\frac{5h}{12}$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{\pi}{3}\left(\frac{5h}{12}\right)^2 h=\frac{\pi}{3}\cdot\frac{25}{144}h^3$

$\frac{d}{dt}[V]=\frac{d}{dt}\left[\frac{\pi}{3}\cdot\frac{25}{144}h^3\right]=\frac{\pi}{\cancel{3}}\cdot\frac{25}{144}\cdot\cancel{3}h^2\cdot\frac{dh}{dt}$

$\frac{dV}{dt}=\left[\frac{25\pi}{144}h^2\right]\frac{dh}{dt}$

$\frac{\frac{dV}{dt}}{\frac{25\pi}{144}h^2}=\frac{dh}{dt}=\frac{10}{\frac{25\pi}{144}\cdot 8^2}$

$=\frac{\cancel{2}\cdot\cancel{5}\cdot 3\cdot\cancel{4}\cdot 3\cdot\cancel{4}}{\cancel{5}\cdot 5\pi\cdot\cancel{4}\cdot\cancel{2}\cdot\cancel{4}\cdot 2}=\boxed{\frac{9}{10\pi}\ \frac{\text{ft}}{\text{min}}}$