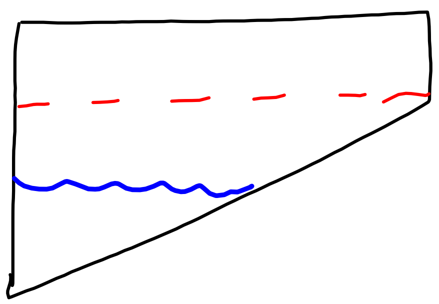
Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates

Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)

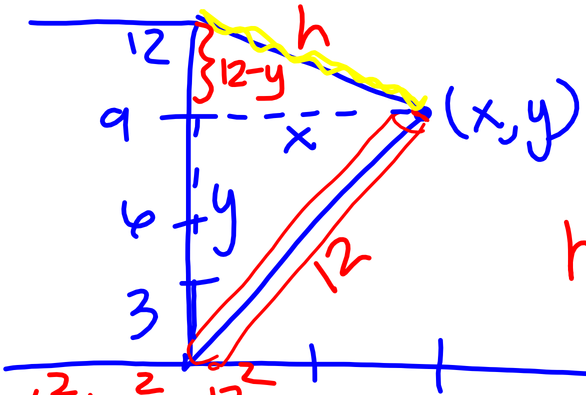
Next week:
3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity



$$\frac{dh}{dt} = -0.2 \text{ m/s}$$

$$\frac{dy}{dt} \text{ \& } \frac{dx}{dt} = ?$$

when $y = 6$



$$h^2 = x^2 + (12-y)^2$$

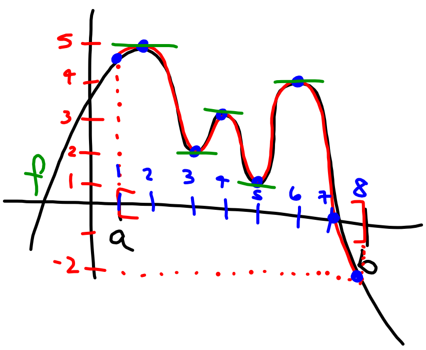
$$x^2 + y^2 = 12^2$$

$$x^2 = 144 - y^2$$

$$h^2 = 144 - y^2 + (12-y)^2$$

3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



relative minima:
$(3,2)$, $(5,1)$

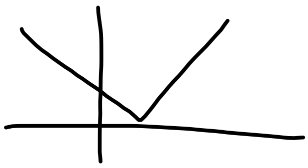
relative maxima:
$(2,5)$, $(4,3)$, $(6,4)$

absolute maximum:
5 @ $(2,5)$

absolute minimum:
-2 @ $(8,-2)$

$f(x)$ can have a relative maximum or minimum when $f'(x) = 0$. or

$f'(x)$ is undefined.



We call such x-values

Critical #'s of f.

3.1 Find the absolute max & min on the closed interval.

28. $h(t) = \dfrac{t}{t-2}$, $[3,5]$

$$h'(t) = \frac{(t-2)\cdot 1 - t(1)}{(t-2)^2} = \frac{-2}{(t-2)^2}$$

critical #'s: $\cancel{t=2}$

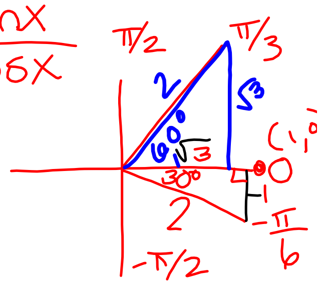
$h(3) = \frac{3}{3-2} = 3 \leftarrow$ abs. max

$h(5) = \frac{5}{5-2} = 5/3 \leftarrow$ abs min

30. $g(x) = \sec x \quad , \left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

Find the absolute max & min on the closed interval.

$g'(x) = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$



critical #'s: 0

$\sec\left(\frac{-\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$\sec(0) = \frac{1}{\cos 0} = 1 \leftarrow$ abs. min

$\sec\left(\frac{\pi}{3}\right) = 2 \leftarrow$ abs. max

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

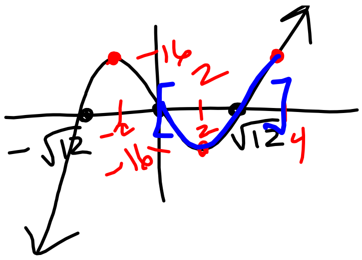
$1 < \sqrt{3} < 2$

$1 > \frac{1}{\sqrt{3}} > \frac{1}{2}$

$2 > \frac{2}{\sqrt{3}} > 1$

22. $f(x) = x^3 - 12x = x(x^2-12) \quad , [0, 4]$

Find the absolute max & min on the closed interval.



$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x-2)(x+2)$

critical #s: $2, \cancel{-2}$

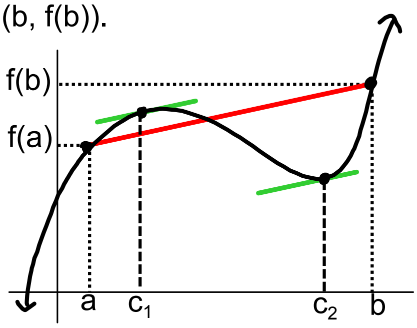
$f(0) = 0$

$f(2) = 8 - 24 = -16 \leftarrow$ abs min

$f(4) = 64 - 48 = 16 \leftarrow$ abs max

3.2 Rolle's Theorem & The Mean Value Theorem

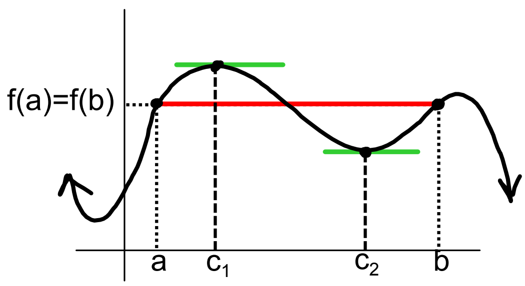
The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



If f is continuous on $[a,b]$ and differentiable on (a,b) then $\exists\, c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}, \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|, \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4, \quad [1,4]$

Is f diff. on (1,4) & cts on [1,4] ? yes

$f(1) = 1 - 5 + 4 = 0$

$f(4) = 16 - 20 + 4 = 0$

$f(1) = f(4)$

yes! Rolle's Thm applies

$f'(x) = 2x - 5$

$2x - 5 = 0$

$x = 5/2$