Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval
Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

3.5 15-31 odd
8.7 11-35 odd
} due Mon

8. Find the limit (if it exists).

$$\lim_{x\to -1}\frac{x^2-x-2}{x^2-1} = \lim_{x\to -1}\frac{(x-2)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{-1-2}{-1-1} = \frac{-3}{-2} = \boxed{\frac{3}{2}}$$

9. Find the limit (if it exists).

$$\lim_{x\to -1}\frac{x^2-9}{x^2-5x+6} = \lim_{x\to 1}\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-2)} = \frac{-1+3}{-1-2} = \boxed{\frac{2}{-3}}$$

10. Find the limit (if it exists).

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x^2-1}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1}\frac{(x+3)-4}{(x^2-1)(\sqrt{x+3}+2)} = \lim_{x\to 1}\frac{\overset{1}{\cancel{x-1}}}{\cancel{(x-1)}(x+1)(\sqrt{x+3}+2)}$$

$$= \frac{1}{(1+1)(\sqrt{1+3}+2)} = \frac{1}{2(2+2)} = \boxed{\frac{1}{8}}$$

3.5 Limits at Infinity

$\lim_{x\to\infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x)=\frac{5x^2-3x+4}{2x^2+5x} \approx \frac{5x^2}{2x^2}=\frac{5}{2}$$

Horizontal asymptote @ $y=5/2$

$\lim_{x\to\infty} f(x)=\frac{5}{2}$ & $\lim_{x\to-\infty} f(x)=\frac{5}{2}$

$$f(x)=\frac{2x-4}{3x^4} \approx \frac{2x}{3x^4}=\frac{2}{3}\cdot\frac{1}{x^3}$$

Horizontal asymptote @ $y=0$

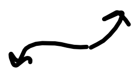
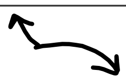
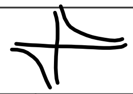
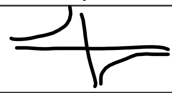
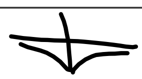
$\lim_{x\to\pm\infty} f(x)=0$

$$f(x) = \frac{2x^7 - 4x^3 - 2}{5x^4 + 1} \approx \frac{2x^7}{5x^4} = \frac{2}{5}x^3$$

$$\lim_{x \to \infty} f(x) = \infty \quad \& \quad \lim_{x \to -\infty} f(x) = -\infty$$

$$f(x) = \frac{2 - 7x^3 + 2x}{1 + x} \approx \frac{-7x^3}{x} = -7x^2$$

$$\lim_{x \to \pm\infty} f(x) = -\infty$$

Ratio of lead terms of $f(x)$	Picture of graph end behavior	$\lim_{x \to -\infty} f(x)$	$\lim_{x \to \infty} f(x)$
$+x^{even}$		∞	∞
$-x^{even}$		$-\infty$	$-\infty$
$+x^{odd}$		$-\infty$	∞
$-x^{odd}$		∞	$-\infty$
c		c	c
$+\frac{1}{x^{odd}}$		0	0
$-\frac{1}{x^{odd}}$		0	0
$+\frac{1}{x^{even}}$		0	0
$-\frac{1}{x^{even}}$		0	0

27. $\lim_{x\to-\infty}\left(\frac{1}{2}x - \frac{4}{x^2}\right)$

$= \lim_{x\to-\infty}\frac{1}{2}x - \lim_{x\to-\infty}\frac{4}{x^2}$

$= -\infty - 0$

$= \boxed{-\infty}$

26. $\lim_{x\to-\infty}\frac{x}{\sqrt{x^2+1}}$

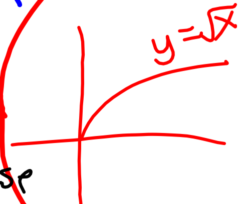
$= \lim_{x\to-\infty}\frac{x}{\sqrt{x^2}}$ ← ratio of lead terms

$= \lim_{x\to-\infty}\frac{x}{|x|}$

$|x| = -x$ because if $x \to -\infty$, $x<0$

$= \lim_{x\to-\infty}\frac{x}{-x}$

$= \lim_{x\to-\infty}(-1) = \boxed{-1}$



$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$\sqrt[2]{(-2)^2}$

$= \sqrt{4} = 2$

$\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x<0 \end{cases}$$

$$\lim_{x\to\infty}\frac{5x-2}{\sqrt{9x^2+3}} = \lim_{x\to\infty}\frac{5x}{\underbrace{\sqrt{9x^2}}_{\sqrt{(3x)^2}}} = \lim_{x\to\infty}\frac{5x}{|3x|}$$

$$= \lim_{x\to\infty}\frac{5x}{3x} = \lim_{x\to\infty}\frac{5}{3} = \boxed{\frac{5}{3}}$$

$$\lim_{x\to-\infty}\left(\qquad\right) = -5/3$$

30. $\lim\limits_{x\to\infty}\dfrac{x-\cos x}{x}$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$= \lim_{x\to\infty}\left(\frac{x}{x} - \frac{\cos x}{x}\right)$$

$$= \lim_{x\to\infty} 1 - \lim_{x\to\infty}\frac{\cos x}{x}$$

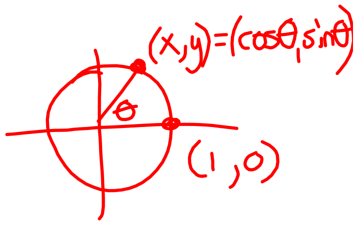
← bounded by $[-1,1]$

$\overline{\quad\infty\quad}$

$$= 1 - 0 = \boxed{1}$$

32. $$\lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos 0 = \boxed{1}$$

$\frac{1}{x} \to 0$ as $x \to \infty$



18. c.

$$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}} = \lim_{x\to\infty} \frac{5}{4}x$$

$$= \boxed{\infty}$$

8.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0}, \ \frac{\infty}{\infty},}$ $0 \cdot \infty, \ 1^{\infty}, \ 0^0, \ and \ \infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form 0/0, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

7.7

12. $\lim_{x\to -1} \frac{x^2-x-2}{x+1}$ $\frac{0}{0}$ $= \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{x+1}} = -1-2 = \boxed{-3}$

$= \lim_{x\to -1} \frac{2x-1}{1} = 2(-1)-1 = \boxed{-3}$

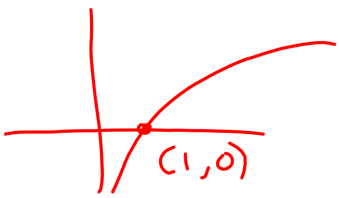
16. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3}$ $\qquad \frac{e^0-1-0}{0^3} = \frac{1-1}{0} = \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2} \qquad \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{e^x}{6x} \qquad \frac{1}{0^+} = \boxed{\infty}$

18. $\lim_{x\to 1} \frac{\ln x^2}{x^2-1} \qquad \frac{0}{0}$

$\ln 1 = 0$



$= \lim_{x\to 1} \frac{\frac{1}{x^2}\cdot \cancel{2x}}{\cancel{2x}}$

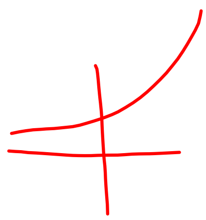
$= \frac{1}{1^2} = \boxed{1}$

20. $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$ $\qquad \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\cos ax \cdot a}{\cos bx \cdot b}$$

$$= \frac{\cos 0 \cdot a}{\cos 0 \cdot b} = \boxed{\frac{a}{b}}$$

28. $\lim_{x\to\infty} \frac{x^2}{e^x}$ $\qquad \frac{\infty}{\infty}$



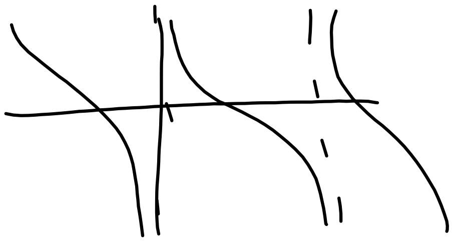
$$= \lim_{x\to\infty} \frac{2x}{e^x} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$$

36. $\lim\limits_{x\to\infty} \dfrac{e^{x/2}}{x}$ $\quad \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{e^{x/2}\cdot\frac{1}{2}}{1} = \boxed{\infty}$

38. $\lim\limits_{x\to 0^+} x^3 \cot x$



$= \lim\limits_{x\to 0^+} \dfrac{x^3}{\tan x}$ $\quad \dfrac{0}{0}$

$= \lim\limits_{x\to 0^+} \dfrac{3x^2}{\sec^2 x} = \dfrac{0}{1} = \boxed{0}$

$\dfrac{0}{1}\cdot\dfrac{\infty}{1}$

$= \dfrac{0}{\frac{1}{\infty}} \qquad = \dfrac{\infty}{\frac{1}{0}}$