Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval
Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity
Due Mon 5/15:
3.5 #15-31 odd - Limits at Infinity
8.7 #15-31 odd - l'Hopital's Rule
Due Fri 5/19:
3.7 #3,5,17,19 - Optimization

3.7 Optimization Problems

x, y

4. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

$S(x,y) = x + 3y$

$S(y) = \frac{192}{y} + 3y = 192y^{-1} + 3y$

$S'(y) = -192y^{-2} + 3$

$-\frac{192}{y^2} + 3 = 0$

$xy = 192$

$x = \frac{192}{y}$ or $y = \frac{192}{x}$

$3 = \frac{192}{y^2}$

$3y^2 = 192$

$y^2 = \frac{192}{3} = 64$

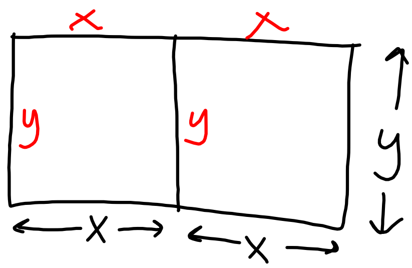
$y^2 = 64$

$y = \pm 8$

$x = \frac{192}{8}$

$= \boxed{24}$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



$A(x,y) = 2xy$

$200 = 4x + 3y \leftarrow$

$200 - 3y = 4x$

$50 - \frac{3}{4}y = x = 50 - \frac{3}{4}\left(\frac{100}{3}\right)$

$\boxed{x = 25\text{ ft}}$

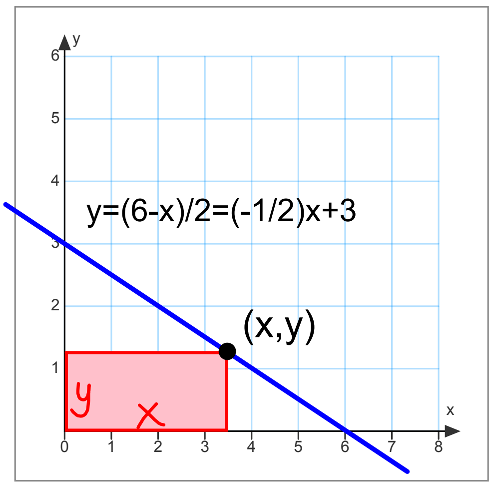
$A(y) = 2\left(50 - \frac{3}{4}y\right)y = 100y - \frac{3}{2}y^2$

$A(y) = 100y - \frac{3}{2}y^2$

$A'(y) = 100 - 3y = 0 \Rightarrow 100 = 3y$

$\boxed{\frac{100}{3}\text{ ft} = y}$

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$A(x,y) = xy$

$A(x) = x\left(-\frac{1}{2}x + 3\right)$

$A(x) = -\frac{1}{2}x^2 + 3x$

$A'(x) = -x + 3$

$-x + 3 = 0$

$\boxed{3 = x}$

$y = \frac{6-x}{2}$

$= \frac{6-3}{2} = \boxed{\frac{3}{2}}$

44. $$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$\ln y = \ln \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$\ln y = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{x}$$

$$\ln y = \lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)$$

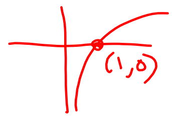
$$\ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$$

$$\ln y = \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}}$$

$$\ln y = 1$$

$$\log a^{p} = p\cdot\log a$$



$$\infty\cdot 0 \qquad \frac{\infty}{1}\cdot\frac{0}{1} = \frac{\infty}{\left(\frac{1}{0}\right)} = \frac{0}{\left(\frac{1}{\infty}\right)}$$

$$\frac{0}{0}$$

$$a^{\log_a x} = x$$

$$\log_a\left(a^{x}\right) = x$$

$$e^{\ln y} = e^{1}$$

$$y = \boxed{e}$$

$$\left[f(g(x))\right]' = f'(g(x))\cdot g'(x)$$

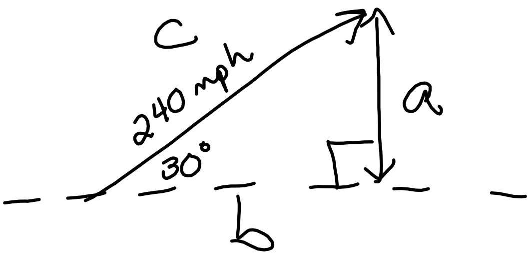
$$\left[\sin(2x)\right]' = \cos 2x \cdot 2$$

$$(\sin x)' = \cos x \cdot 1$$

$$f(x) = [2g(x)]h(x)$$

$$f'(x) = 2g'(x) \cdot h(x) + 2g(x) \cdot h'(x)$$

$$f'(3) = 2g'(3) \cdot h(3) + 2g(3) + h'(3)$$



$$240 = \frac{dc}{dt}$$

$$\frac{da}{dt} = ?$$

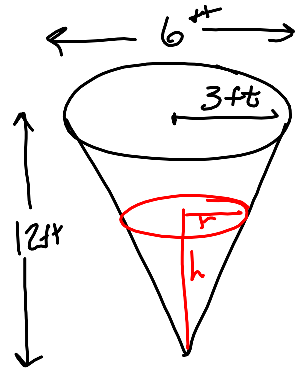
$$\sin 30^\circ = \frac{a}{c}$$

$$\frac{1}{2} = \frac{a}{c} \qquad \boxed{2a = c}$$

$$\frac{\cancel{2}\frac{da}{dt}}{\cancel{2}} = \frac{\frac{dc}{dt}}{2}$$

$$\frac{da}{dt} = 120 \text{ mph}$$

$10\,\frac{ft^3}{min} = \frac{dV}{dt}$; $\frac{dh}{dt} = ?$ when $h = 8\,ft$



$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{3}{12}$$

$$r = \frac{h}{4}$$

$$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 \cdot h = \frac{\pi}{3 \cdot 16} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{3}\cdot 16} \cdot \cancel{3}h^2 \cdot \frac{dh}{dt}$$

$$\frac{10}{\frac{\pi}{16}\cdot 8^2} = \frac{\frac{dV}{dt}}{\frac{\pi}{16}h^2} = \frac{dh}{dt}$$

$$\frac{\cancel{16}\cdot \cancel{10}^{\,5}}{\pi \cdot \cancel{8} \cdot \cancel{8}\ \cancel{4}\ 2}$$

$$= \boxed{\frac{5}{2\pi}\ ft/min}$$