Homework for Test #1

- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31

3.9

15. $y = x\sqrt{1-x^2} = x \cdot (1-x^2)^{1/2}$

$\frac{s^2}{s^2} + \frac{c^2}{s^2} \quad \frac{1}{s^2}$

$1 + \cot^2 = \csc^2$

$\cdot\, dx$

$dy = 1 \cdot (1-x^2)^{1/2} + x \cdot \frac{1}{\cancel{2}}(1-x^2)^{-1/2} \cdot (-\cancel{2}x)$

$= \left(\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right) dx$

$= \frac{1-x^2-x^2}{\sqrt{1-x^2}} dx$

$= \frac{1-2x^2}{\sqrt{1-x^2}} dx$

17.

$dy = \left(2 + 2\cot x \frac{\csc^2 x}{1+\cot^2 x}\right) dx$

3.9 #50

Use differentials to approximate $\tan(0.05)$.

we know $\tan(0) = 0$

$0.05 = 0 + \Delta x$

$f(c + \Delta x) \approx f(c) + f'(x)dx$

$f(x) = \tan x$; $c = 0$; $\Delta x = dx = 0.05$

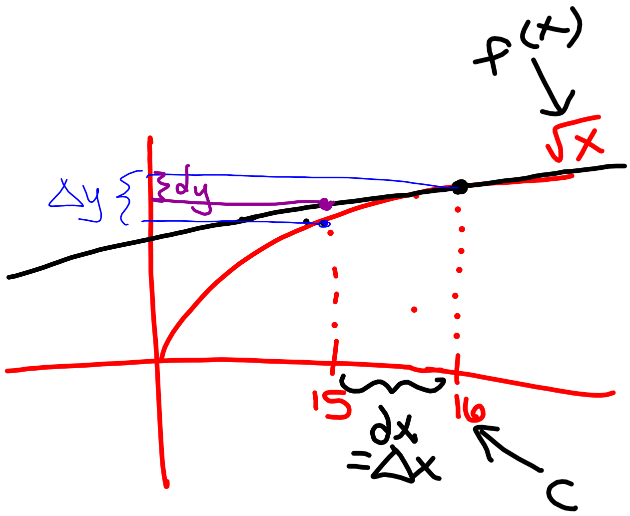
$f'(x) = \sec^2 x$ $\qquad \tan(0) = 0$

$\tan(0 + 0.05) \approx 0 + \sec^2(0) \cdot 0.05$

$\tan 0.05 \approx \boxed{0.05}$

Why does the differential give us a good approximation for the actual change in y?

locally, functions behave linearly



$\sqrt{15} = \sqrt{16 - 1}$

$\approx \sqrt{16} + \frac{1}{2\sqrt{16}} \cdot (-1)$

$y - y_1 = m(x - x_1)$

$y \approx \underset{f(c)}{y_1} + \underset{f'(c)}{m}\underbrace{(x - x_1)}_{\Delta x}$

4.1 Antiderivatives

$F(x) = 5x^4$

$f(x) = x^5$ $\qquad [f(x)]' = 5x^4$

particular solution

General solution: $x^5 + C$

Find a general solution and a particular solution to the differential equation.

56. $g'(x) = 6x^2$, $g(0) = -1$

$y' = 6x^2$

$y = 2x^3 + C$

$g(x) = 2x^3 + C$ general solution

$-1 = 2(0)^3 + C$

$-1 = C$

particular Solution

$g(x) = 2x^3 - 1$

$$y = F(x)$$

$$\frac{dy}{dx} = f(x)$$

$$\int dy = \int f(x)\,dx$$

$$y = \int f(x)\,dx = F(x) + C$$

antiderivative = indefinite integral

18. $\int (4x^3 + 6x^2 - 1)\,dx$

$= \boxed{x^4 + 2x^3 - x + C}$

24. $\int (\sqrt[4]{x^3} + 1)\,dx = \int (x^{3/4} + 1)\,dx$

$= \boxed{\frac{4}{7}x^{7/4} + x + c}$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$$

$$x^{-1} = \frac{1}{x}$$

28. $$\int \frac{x^2+2x-3}{x^4}dx = \int\left(x^{-2}+2x^{-3}-3x^{-4}\right)dx$$

$$= \boxed{-x^{-1}-x^{-2}+x^{-3}+C}$$

$$= \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$$

$$= \boxed{\frac{-x^2-x+1}{x^3} + C}$$

38. $$\int\left(\theta^2+\sec^2\theta\right)d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$$

42. $$\int \frac{\cos x}{1-\cos^2 x}dx = \int \frac{\cos x}{\sin^2 x}dx$$

$$= \int \frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}dx = \int \csc x \cot x \, dx$$

$$= \boxed{-\csc x + C}$$

40. $\int \sec y(\tan y - \sec y)\,dy$

$= \int (\sec y \tan y - \sec^2 y)\,dy$

$= \boxed{\sec y - \tan y + C}$

58. $f'(s) = 6s - 8s^3$, $f(2) = 3$

$\frac{df}{ds} = 6s - 8s^3$

$df = (6s - 8s^3)\,ds$

$\int df = \int (6s - 8s^3)\,ds$

$f(s) = 3s^2 - 2s^4 + C$ ← general solution

$3 = 3(2)^2 - 2(2)^4 + C$

$3 = 12 - 32 + C$

$23 = C$

particular Solution : $\boxed{f(s) = 3s^2 - 2s^4 + 23}$

62. $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

$f'(x) = -\cos x + c_1$

$1 = -\cos(0) + c_1$
$2 = c_1$

$f(x) = -\sin x + \cancel{c_1}x + c_2$
2

particular solution:

$\boxed{f(x) = -\sin x + 2x + 6}$

$6 = -\sin(0) + 2(0) + c_2$
$6 = c_2$

$s(t) =$ position s m

$v(t) = s'(t) =$ velocity $\frac{\Delta s}{\Delta t}$ m/s

$a(t) = v'(t) = s''(t) =$ acceleration

$\frac{\Delta s}{\Delta t} / \Delta t$ m/s^2

$a(t) = -32 \text{ ft/s}^2$

$v(t) = -32t + v_0$

$s(t) = -16t^2 + v_0 t + s_0$

72. 1600 m $\qquad a = -9.8 \text{ m/s}^2$

$a(t) = -9.8$

$v(t) = -9.8t + v_0$, $v_0 = 0$ (rock is dropped)

$\boxed{s(t) = -4.9t^2 + 1600}$

$0 = -4.9t^2 + 1600$

$4.9t^2 = 1600$

$t^2 = \dfrac{16000}{49}$

$\boxed{t = \dfrac{40\sqrt{10}}{7} \text{ s}}$

80. $a(t) = \cos t$, $t > 0$

@ $t = 0$; position is $x = 3$ $s(0) = 3$

$v(0) = 0$

a) find velocity & position functions

b) find value(s) of t for which the particle is at rest.

$0 = \sin(0) + C_1$

$0 = C_1$

$v(t) = \sin t + 0$

$s(t) = -\cos t + 4$

$3 = -\cos(0) + C_2$

$4 = C_2$

$\sin t = 0$

$t = 0, \pi, 2\pi, 3\pi, \ldots \pi k, k > 0$