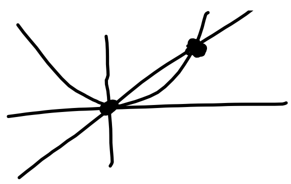
Homework for Test #1

- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31

4.1

#83.



$a_c(t) = 6\ ft/s^2 \rightarrow V_c(t) = 6t \rightarrow S_c(t) = 3t^2$

$V_T(t) = 30\ ft/s \rightarrow S_T(t) = 30t$

$3t^2 = 30t$ $\quad t = 10s \quad s(10) = \boxed{300\ ft}$

$3t^2 - 30t = 0$ $\quad V_c(10) = 6(10) = \boxed{60\ ft/s}$

$3t(t-10) = 0$

4.2 Area

$\int f(x)\,dx$

Sigma Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

Summation Formulas

1. $\sum_{i=1}^{n} c = nc$
2. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$

Use sigma notation to write the sum.

8. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\dots+\frac{5}{1+15} = \sum_{i=1}^{15}\frac{5}{1+i} = \sum_{n=1}^{15}\frac{5}{1+n}$

14. $\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{0}{n}\right)^2}+\dots+\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{n-1}{n}\right)^2} = \sum_{i=0}^{n-1}\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{i}{n}\right)^2}$

Use summation properties to evaluate the sum.

20. $\sum_{i=1}^{10} i(i^2+1) = \sum_{i=1}^{10}(i^3+i)$

$= \sum_{i=1}^{10} i^3 + \sum_{i=1}^{10} i$

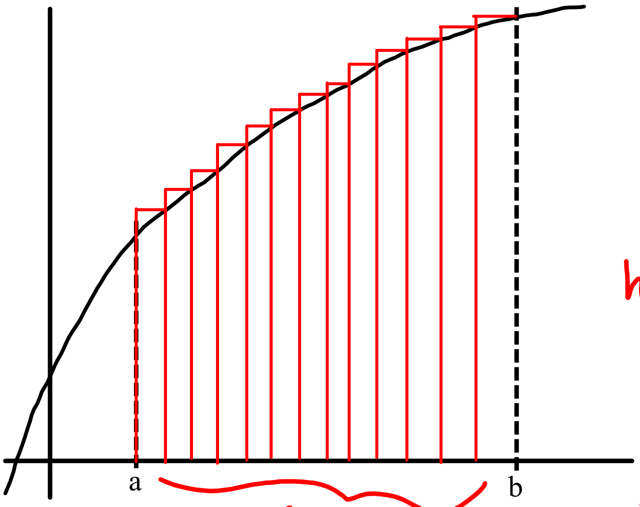
$= \frac{10^2(10+1)^2}{4} + \frac{10(10+1)}{2}$

$= \frac{100\cdot 121}{4} + \frac{110}{2}$

$= 25\cdot 121 + 55$

$= \boxed{3080}$

4.2 Area



n equal intervals

width of each is $\frac{b-a}{n}$

Divide the interval [a,b] into n equal subintervals, each of width (b-a)/n.

Here, the height of a rectangle is determined by the right endpoint of each subinterval; this is called an upper sum.

~~The area of each rectangle will be:~~

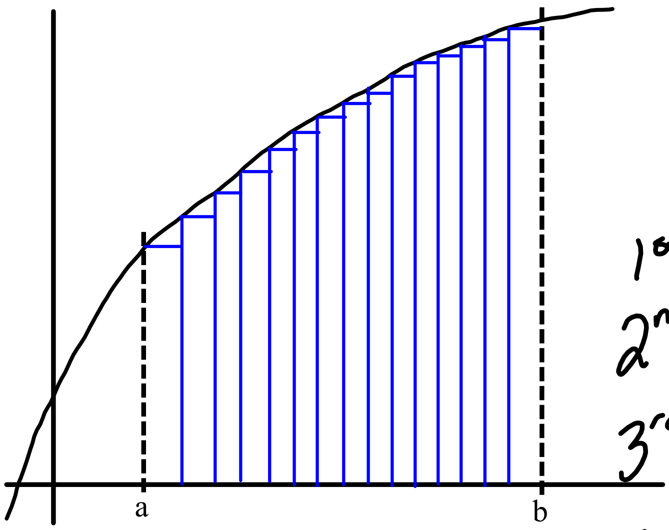
height of 1^{st} rectangle is $f\left(a+1\cdot\frac{b-a}{n}\right)$

& hence area is $\frac{b-a}{n}\cdot f\left(a+\frac{b-a}{n}\right)$

2^{nd} rectangle height: $f\left(a+2\cdot\frac{b-a}{n}\right)$

area: $\frac{b-a}{n}\cdot f\left(a+2\cdot\frac{b-a}{n}\right)$

i^{th} area: $\boxed{\frac{b-a}{n}\cdot f\left(a+i\cdot\frac{b-a}{n}\right)}$



Here, the height of a rectangle is determined by the left endpoint of each subinterval; this is called a lower sum.

The area of each rectangle will be:

1^{st}: $\frac{b-a}{n}\cdot f(a)$

2^{nd}: $\frac{b-a}{n}\cdot f\left(a+\frac{b-a}{n}\right)$

3^{rd}: $\frac{b-a}{n}\cdot f\left(a+2\cdot\frac{b-a}{n}\right)$

i^{th}: $\frac{b-a}{n}\cdot f\left(a+(i-1)\frac{b-a}{n}\right)$

Lower sum: $s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$

Upper sum: $S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$

$f(m_i)$ = minimum function value in an interval

$f(M_i)$ = Maximum function value in an interval

$\Delta x = \frac{b-a}{n}$ $s(n) \le S(n)$

Area of the region bounded by the graph of f, the x-axis, & the lines $x=a$ & $x=b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \quad x_{i-1} \le c_i \le x_i$$

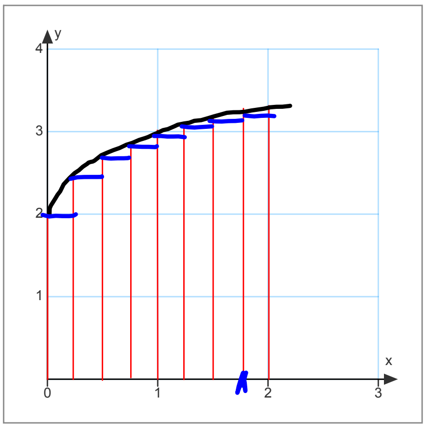
any x-value in each of the n intervals

where $\Delta x = \frac{b-a}{n}$.

28. $y=\sqrt{x}+2$

Use upper sum and lower sum to approximate the area under the curve.

$n=8;\ a=0;\ b=2 \qquad \Delta x=\frac{2-0}{8}=\frac{1}{4}$



upper sum:

$$\text{area}_i=\overset{1/4}{\frac{b-a}{n}}\cdot f\Big(\overset{0}{a}+i\cdot\overset{1/4}{\frac{b-a}{n}}\Big)$$

$$\sum_{i=1}^{8}\frac{1}{4}\left(\sqrt{\frac{i}{4}}+2\right)=\sum_{i=1}^{8}\frac{\sqrt{i}}{8}+\sum_{i=1}^{8}\frac{1}{2}$$

$$=\frac{1}{8}\left(\sqrt{1}+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}+\sqrt{7}+\sqrt{8}\right)+4$$

$$=\boxed{6.04}$$

lower sum:

$$\frac{1}{4}(\sqrt{0}+2)+\frac{1}{4}\left(\sqrt{1/4}+2\right)+\frac{1}{4}\left(\sqrt{2/4}+2\right)+\frac{1}{4}\left(\sqrt{3/4}+2\right)+$$

$$+\frac{1}{4}\left(\sqrt{4/4}+2\right)+\frac{1}{4}\left(\sqrt{5/4}+2\right)+\frac{1}{4}\left(\sqrt{\frac{6}{4}}+2\right)+\frac{1}{4}\left(\sqrt{\frac{7}{4}}+2\right)$$

$$=\boxed{\quad}$$