Homework for Test #1

- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31

Test #1 - Wed, 11/18?

4.2 #17

$$\sum_{i=1}^{20}(i-1)^2 = \sum i^2 - 2i + 1$$

$$= \sum i^2 - \sum 2i + \sum 1$$

$$= \sum i^2 - 2\sum i + \sum 1$$

$$= \frac{20(20+1)(2\cdot 20+1)}{6} - 2\cdot\frac{20(20+1)}{2} + 20$$

$$= \frac{20(21)(41)}{6} - 20(21) + 20$$

$$= \frac{20(21)(41) - 6(20)(21) + 6(20)}{6}$$

$$= \boxed{2470}$$

$\lim_{n\to\infty} S(n)$

32. $S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$\approx \frac{64 \cdot \cancel{2} \cdot \cancel{n^3}}{\cancel{6}_3 \cdot \cancel{n^3}}$

$= \boxed{\frac{64}{3}}$

rewrite without summation notation

36. $\sum_{j=1}^{n} \frac{4j+3}{n^2} = \frac{1}{n^2}\sum_{j=1}^{n} 4j+3$

$= \frac{1}{n^2}\left(4\sum_{j=1}^{n} j + \sum_{j=1}^{n} 3\right)$

$= \boxed{\frac{1}{n^2}\left(4 \cdot \frac{n(n+1)}{2} + 3n\right)}$

$= \frac{2n(n+1)+3n}{n^2} = \frac{2n^2+2n+3n}{n^2} = \boxed{\frac{2n^2+5n}{n^2}}$

44. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(1+\frac{2i}{n}\right)^3 \left(\frac{2}{n}\right)$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(1 + 3\cdot\frac{2i}{n} + 3\cdot\frac{4i^2}{n^2} + \frac{8i^3}{n^3}\right)\cdot\frac{2}{n}$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\frac{2}{n} + \frac{12i}{n^2} + \frac{24i^2}{n^3} + \frac{16i^3}{n^4}\right)$

$= \lim\limits_{n\to\infty} \left(\frac{2}{n}\sum\limits_{i=1}^{n} 1 + \frac{12}{n^2}\sum\limits_{i=1}^{n} i + \frac{24}{n^3}\sum\limits_{i=1}^{n} i^2 + \frac{16}{n^4}\sum\limits_{i=1}^{n} i^3\right)$

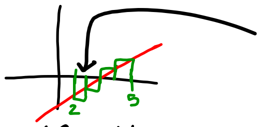
$= \lim\limits_{n\to\infty} \left(\frac{2}{n}\cdot n + \frac{12}{n^2}\cdot\frac{n(n+1)}{2} + \frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right)$

$= \lim\limits_{n\to\infty} \left(\frac{2n}{n} + \frac{12n^2}{2n^2} + \frac{48n^3}{6n^3} + \frac{16n^4}{4n^4}\right)$

$= 2 + 6 + 8 + 4$

$= \boxed{20}$

48. $y = 3x - 4$, $[2, 5]$



$\text{Area} = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(c_i)\Delta x$, $x_i \le c_i \le x_{i+1}$,

$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[3\left(2+\frac{i\cdot 3}{n}\right) - 4\right]\cdot\frac{3}{n}$

right-hand endpoint of i^{th} interval

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[6 + \frac{9i}{n} - 4\right]\cdot\frac{3}{n}$ $\quad \left(2+\frac{9i}{n}\right)$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\frac{6}{n} + \frac{27i}{n^2}\right)$

$= \lim\limits_{n\to\infty} \left(\frac{6}{n}\sum\limits_{i=1}^{n} 1 + \frac{27}{n^2}\sum\limits_{i=1}^{n} i\right)$

$= \lim\limits_{n\to\infty} \left(\frac{6}{n}\cdot n + \frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right)$

$= \lim\limits_{n\to\infty} \left(\frac{6n}{n} + \frac{27n^2}{2n^2}\right) = 6 + \frac{27}{2} = \boxed{\frac{39}{2}}$

4.3 Riemann Sums & Definite Integrals

$$\sum_{i=1}^{n} f(c_i)\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i ,$$

where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ is called the Riemann Sum of f.

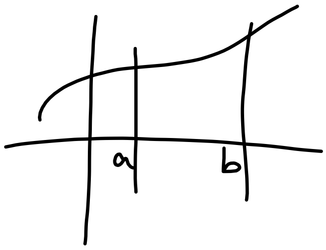
$$\lim_{\substack{\Delta x \to 0 \\ (n \to \infty)}} \sum_{i=1}^{n} f(\overset{a + i \cdot \frac{b-a}{n}}{c_i}) \underset{\left(\frac{b-a}{n}\right)}{\Delta x} = \int_a^b f(x)\,dx$$

called the definite integral of f from a to b.

Properties

If $f(a)$ is defined,

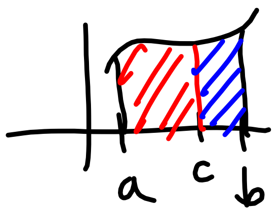
$$\int_a^a f(x)\,dx = 0$$



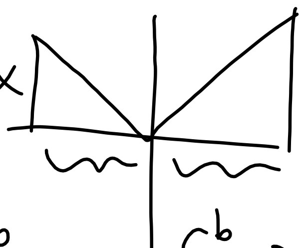
If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$,

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

typo on p. 272