Homework for Test #1

- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31

Test #1 - Wed, 11/18

Find the differential dy.

1. $y = 5x^2 + 9$

$dy = (10x)dx$

2. $y = 2x - \csc x$

$dy = (2 + \csc x \cot x)dx$

Find the antiderivative.

3. $\int (x^4 - 3)dx$

$\frac{1}{5}x^5 - 3x + C$

4. $\int (\sec^2\theta + \cos\theta)\, d\theta$

$\tan\theta + \sin\theta + C$

Approximate the function value using differentials. *Hint:* $f(c+\Delta x) \approx f(c) + f'(c)\Delta x$

5. $\sqrt{101}$ $= \sqrt{100+1}$ $= 10 + \frac{1}{20}(1) = \boxed{\frac{201}{20}}$ $\approx$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$c = 100$

$\Delta x = 1$

$f(c) = \sqrt{100} = 10$

$f'(c) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

56. $y = x^2 - x^3$ $[-1, 0]$

$= x^2(1-x)$

$\frac{b-a}{n} = \frac{0-(-1)}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(-1+\frac{i}{n}\right)^2 - \left(-1+\frac{i}{n}\right)^3\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[1 - \frac{2i}{n} + \frac{i^2}{n^2} - \left(-1 + \frac{3i}{n} - \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{1}{n} - \frac{2i}{n^2} + \frac{i^2}{n^3} + \frac{1}{n} - \frac{3i}{n^2} + \frac{3i^2}{n^3} - \frac{i^3}{n^4}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2}{n} - \frac{5i}{n^2} + \frac{4i^2}{n^3} - \frac{i^3}{n^4}\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\sum 1 - \frac{5}{n^2}\sum i + \frac{4}{n^3}\sum i^2 - \frac{1}{n^4}\sum i^3\right]$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\cdot n - \frac{5}{n^2}\cdot\frac{n(n+1)}{2} + \frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right)$$

$$= 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4}$$

$$= \frac{24}{12} - \frac{30}{12} + \frac{16}{12} - \frac{3}{12}$$

$$= \boxed{\frac{7}{12}}$$

4.3 Riemann Sums & Definite Integrals

$\sum_{i=1}^{n} f(c_i)\Delta x_i$, $x_{i-1} \le c_i \le x_i$,

where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ is called the Riemann Sum of f.

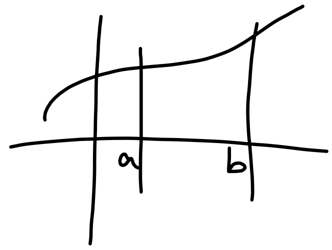
$$\lim_{\substack{\Delta x \to 0 \\ (n \to \infty)}} \sum_{i=1}^{n} f(\underbrace{c_i}_{a + i \cdot \frac{b-a}{n}}) \underbrace{\Delta x}_{\left(\frac{b-a}{n}\right)} = \int_a^b f(x)\,dx$$

called the definite integral of f from a to b.

Properties

If $f(a)$ is defined,

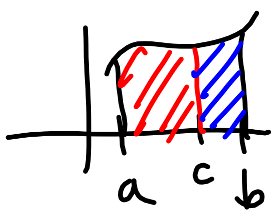
$$\int_a^a f(x)\,dx = 0$$



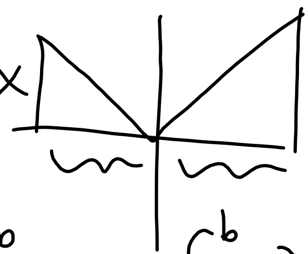
If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$,

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

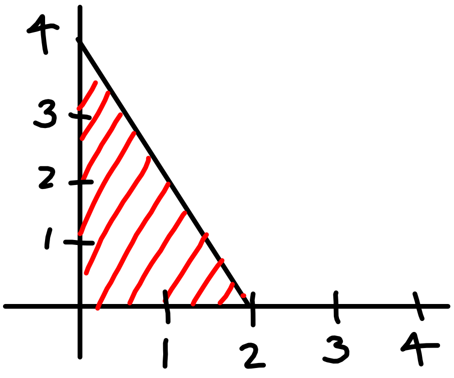
typo on p. 272

8. $$\int_{-1}^{2} (3x^2+2)\,dx$$

$$\frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(3\left(-1+\frac{3i}{n}\right)^2 + 2\right)$$

14. $f(x) = 4 - 2x = -2x + 4$



$$\int_0^2 f(x)\,dx$$

$$= \frac{1}{2}(2)(4)$$

$$= \boxed{4}$$

40. $\int_2^4 x^3\,dx = 60$; $\int_2^4 x\,dx = 6$; $\int_2^4 dx = 2$

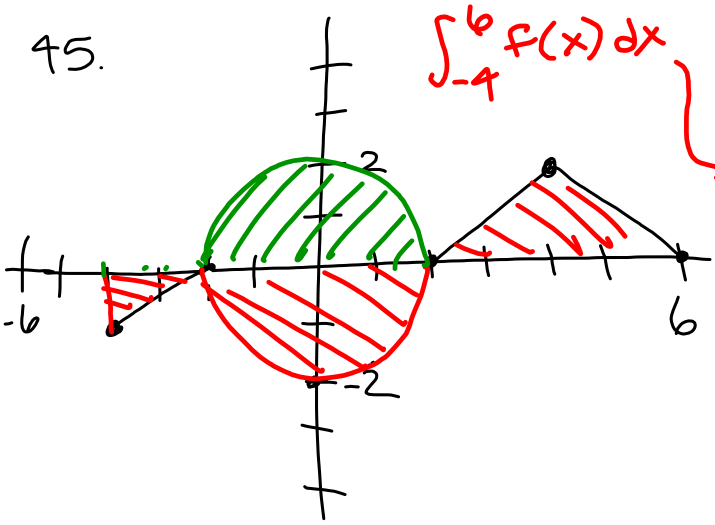
$$\int_2^4 (6 + 2x - x^3)\,dx$$

$$= \int_2^4 6\,dx + \int_2^4 2x\,dx - \int_2^4 x^3\,dx$$

$$= 6\int_2^4 dx + 2\int_2^4 x\,dx - \int_2^4 x^3\,dx$$

$$= 6 \cdot 2 + 2 \cdot 6 - 60 = \boxed{-36}$$

45.



$\int_{-4}^{6} f(x)\,dx$

$$= -\frac{1}{2}(2)(1) - \frac{\pi(2)^2}{2} + \frac{1}{2}\cdot 4\cdot 2$$

$$= -1 - 2\pi + 4 = \boxed{3 - 2\pi}$$

b) $\int_{2}^{6} f(x)\,dx = 4$

$$\int_{-2}^{2} |f(x)|\,dx$$

$$= \frac{\pi(2)^2}{2} = 2\pi$$

4.4 The Fundamental Theorem of Calculus

continuous

If f is $\boxed{\text{cts}}$ on $[a,b]$ and F is antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

10. $\int_1^3 (3x^2+5x-4)\,dx$

$= \left[x^3+\frac{5}{2}x^2-4x\right]\Big|_{x=1}^{3}$

$= \left(3^3+\frac{5}{2}\cdot 3^2-4\cdot 3\right)-\left(1^3+\frac{5}{2}\cdot 1^2-4\cdot 1\right)$

$= 27+\frac{45}{2}-12-1-\frac{5}{2}+4$

$= \boxed{38}$

24. $\int_1^4 (3-|x-3|)\,dx$

$|x-3| = \begin{cases} x-3, & x \geq 3 \\ -(x-3), & x<3 \end{cases}$

$= \int_1^3 \left(3-\left[-(x-3)\right]\right)dx+\int_3^4 \left(3-(x-3)\right)dx$

$= \int_1^3 (x)\,dx + \int_3^4 (6-x)\,dx$

$= \frac{x^2}{2}\Big|_1^3 + \left(6x-\frac{x^2}{2}\right)\Big|_3^4$

$= \frac{3^2}{2}-\frac{1^2}{2}+\left(6\cdot 4-\frac{4^2}{2}\right)-\left(6\cdot 3-\frac{3^2}{2}\right)$

$= \frac{13}{2}$