## Homework for Test #1

- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31

Test #1 - Wed, 11/18

4.3

#45 (f)

$$\int_{-4}^{6}[f(x)+2]\,dx = \underbrace{\int_{-4}^{6} f(x)\,dx}_{\text{use areas of shapes}} + \int_{-4}^{6} 2\,dx$$

$$2x\Big|_{-4}^{6}$$

$$12-(-8)$$

$f''(x) = 6x - 6$ ; $(2, 1)$

$3x - y - 5 = 0$

$\boxed{3}x - 5 = y \longrightarrow f'(2) = 3$

$f'(x) = 3x^2 - 6x + C_1 \quad {=}3$

$3 = 3(2)^2 - 6(2) + C_1$

$3 - 12 + 12 = C_1$

$f(x) = x^3 - 3x^2 + 3x + C_2$

$1 = 2^3 - 3(2)^2 + 3(2) + C_2$

$1 - 8 + 12 - 6 = C_2$

$-1 = C_2$

$f(x) = x^3 - 3x^2 + 3x - 1$

$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (c_i^2 + 5)\Delta x_i$ $[2, 3]$

$= \int_2^3 (x^2 + 5)\, dx$

$$\int_1^3 (2x+5)\,dx \qquad a=1,\ b=3 \qquad \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

x-value ~~point~~ in ith interval

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2\left(1+\frac{2i}{n}\right)+5\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2+\frac{4i}{n}+5\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(7+\frac{4i}{n}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{14}{n}+\frac{8i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{14}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left(\frac{14}{n}\cdot n + \frac{8}{n^2}\cdot\frac{n(n+1)}{2}\right) = 14+4 = \boxed{18}$$

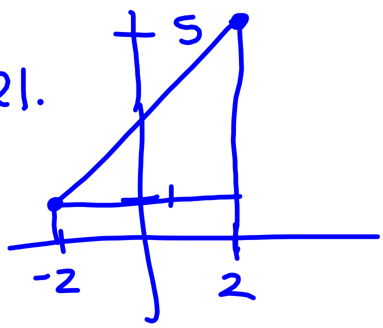
FTC check

$$x^2+5x\Big|_1^3$$

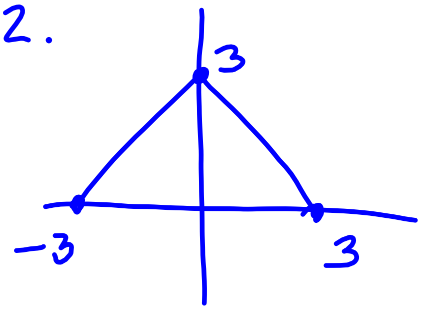
$$(9+15)-(1+5)$$

$$24-6=\boxed{18}$$

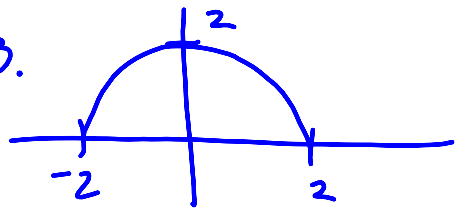
21.




22.




23.




$$x^2+y^2=r^2$$

$$y^2=r^2-x^2 \qquad y=\pm\sqrt{r^2-x^2}$$

32. $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\,dt$

$= t^2 + \sin t \Big|_{-\pi/2}^{\pi/2}$

$= \cancel{\left(\frac{\pi}{2}\right)^2} + \sin\frac{\pi}{2} - \left(\cancel{\left(\frac{-\pi}{2}\right)^2} + \sin\left(\frac{-\pi}{2}\right)\right)$

$= 1 - (-1) = \boxed{2}$

4.3 $\int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

$\int_a^c = \int_a^b + \int_b^c$ $\int_a^b = -\int_b^a$

44. Given $\int_{-1}^{1} f(x)dx = 0$ & $\int_0^1 f(x)dx = 5$

$\int_{-1}^{0} f(x)dx + 5 = 0$

(a) $\int_{-1}^{0} f(x)dx = \boxed{-5}$

(c) $\int_{-1}^{1} 3f(x)dx = 3 \cdot 0 = 0$

(b) $\int_0^1 f(x)dx - \int_{-1}^{0} f(x)dx$

$5 - (-5)$

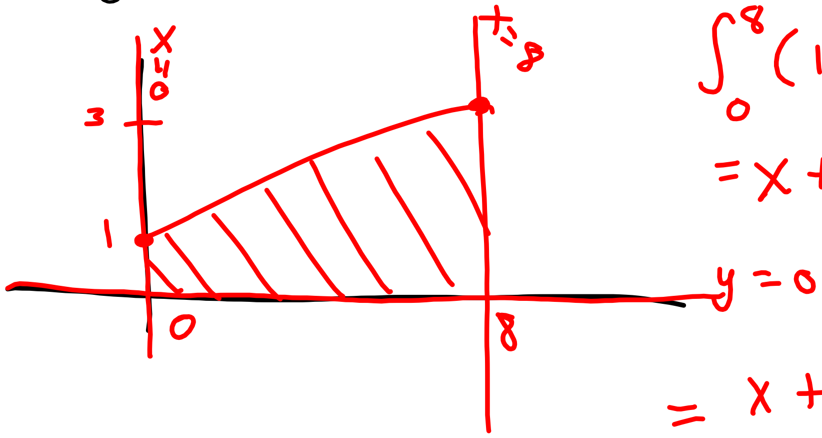
$= 10$

(d) $\int_0^1 3f(x)dx = 3 \cdot 5 = 15$

(e) $\int_1^0 f(x)dx = -\int_0^1 f(x)dx = -5$

4.4 find area of region bounded by...

42. $y = 1 + \sqrt[3]{x}$ , $x=0$, $x=8$, $y=0$




$x^{1/3} = \sqrt[3]{x}$

$$\int_0^8 \left(1+\sqrt[3]{x}\right) dx$$

$$= x + \frac{3}{4}x^{4/3} \Big|_0^8$$

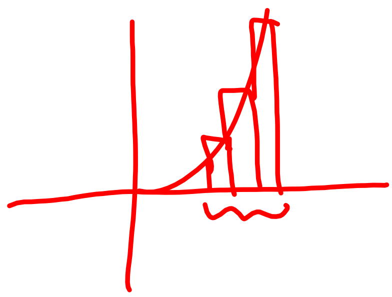
$$= x + \frac{3}{4}\left(\sqrt[3]{x}\right)^4 \Big|_0^8$$

$$= 8 + \frac{3}{4}\left(\sqrt[3]{8}\right)^4 - 0$$

$$= 20$$

4.3 calculate using limit def.

6. $\int_1^3 3x^2 dx$



$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left( \frac{2}{n} \cdot 3 \underbrace{\left(1 + \frac{2i}{n}\right)}_{a + \frac{b-a}{n}\cdot i}{}^2 \right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \left( \frac{6}{n}\sum 1 + \frac{24}{n^2}\sum i + \frac{24}{n^3}\sum i^2 \right.$$

$$= \lim_{n\to\infty} \left( \frac{6}{n}\cdot n + \frac{24}{n^2}\cdot\frac{n(n+1)}{2} + \frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} \right)$$

$$= 6 + 12 + 8 = 26$$

4.2

40. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$\lim_{n\to\infty} \frac{4}{n^2} \sum_{i=1}^{n} i$

$\lim_{n\to\infty} \frac{4}{n^2} \cdot \frac{n(n+1)}{2} = \boxed{2}$

60. $f(y) = 4y - y^2$ , $1 \le y \le 2$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[4\left(1+\frac{i}{n}\right) - \left(1+\frac{i}{n}\right)^2\right]$

$= \int_1^2 (4y - y^2)\,dy = 2y^2 - \frac{1}{3}y^3 \Big|_1^2$