4.4 #45-51 odd; 75-91 odd
→ 4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all

4.4 – The Fundamental Theorem of Calculus

1st Fundamental Theorem of Calculus:
If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Mean Value Theorem for Integrals:
If a function f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a)$$

Recall the Mean Value Theorem:
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Average value of a function on an interval:

If f is integrable on the closed interval $[a, b]$, then the average value of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

The Second Fundamental Theorem of Calculus:

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$$\boxed{\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)}$$

$$\frac{d}{dx}\int_5^{\cos x} t^3\,dt = (\cos^3 x)(-\sin x)$$

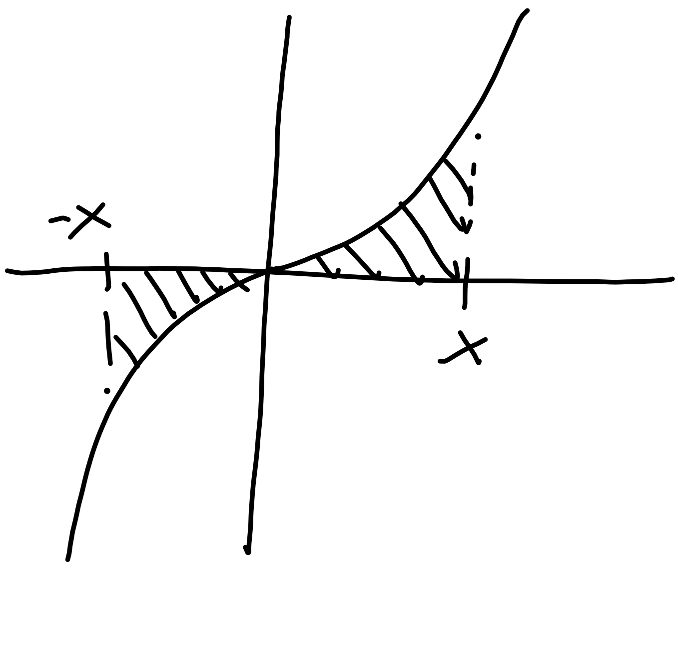
88. $F(x) = \int_{-x}^{x} t^3 dt = \int_{-x}^{a} t^3 dt + \int_{a}^{x} t^3 dt$

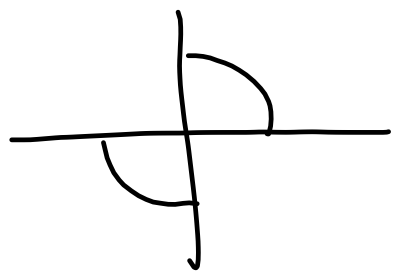
$$\boxed{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx}$$

$F(x) = -\int_{a}^{-x} t^3 dt + \int_{a}^{x} t^3 dt$

$F'(x) = -(-x)^3(-1) + x^3$

$= -x^3 + x^3 = 0$





4.5 Integration by Substitution

12. $\int x^2(x^3+5)^4\,dx = \int (x^3+5)^4 \cdot x^2\,dx$

Let $u = x^3+5$

$\frac{du}{3} = \frac{\cancel{3}x^2\,dx}{\cancel{3}}$

$\frac{1}{3}du = x^2\,dx$

$= \int u^4\left(\frac{1}{3}du\right)$

$= \frac{1}{15}u^5 + C$

$= \boxed{\frac{1}{15}(x^3+5)^5 + C}$

22. $\int \frac{x^2}{(16-x^3)^2}\,dx = \int \frac{du}{-3u^2}$

$u = 16-x^3$

$\frac{du}{-3} = \frac{\cancel{-3}x^2\,dx}{\cancel{-3}}$

$= \int -\frac{1}{3}u^{-2}\,du$

$= \frac{1}{3}u^{-1} + C$

$= \frac{1}{3}(16-x^3)^{-1} + C$

$= \boxed{\frac{1}{3(16-x^3)} + C}$

4.5

50. $\int \sqrt{\tan x}\ \sec^2 x\ dx = \int u^{1/2}\,du$

$u = \tan x$

$du = \sec^2 x\,dx$

$u = (\tan x)^{1/2}$

$du = \frac{1}{2}(\tan x)^{-1/2} \cdot \sec^2 x\, dx$

$= \frac{2}{3}u^{3/2} + c$

$= \boxed{\frac{2}{3}(\tan x)^{3/2} + c}$

54. $\int \csc^2\left(\frac{x}{2}\right) dx$

52. $\int \frac{\sin x}{\cos^3 x}\,dx = \int \frac{-du}{u^3} = \int -u^{-3}\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= \frac{1}{2}u^{-2} + c$

$= \frac{1}{2}(\cos x)^{-2} + c$

$= \boxed{\frac{1}{2\cos^2 x} + c}$

58. $\int x\sqrt{2x+1}\,dx = \int \left(\frac{1}{2}u^{2/2}-\frac{1}{2}\right)\cdot u^{1/2}\cdot\frac{1}{2}du$

$\sqrt{u}$

$\sqrt{u} = u^{1/2}$

$u = 2x+1 \longrightarrow u-1 = 2x$

$du = 2dx \qquad \frac{1}{2}u-\frac{1}{2} = x$

$\frac{1}{2}du = dx$

$= \int \left(\frac{1}{4}u^{3/2} - \frac{1}{4}u^{1/2}\right)du$

$= \frac{1}{10}u^{5/2} - \frac{1}{6}u^{3/2} + C$

$= \boxed{\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C}$

62. $\int \frac{2x+1}{\sqrt{x+4}}dx = \int \frac{2u-7}{\sqrt{u}}du = \int u^{-1/2}(2u-7)du$

$u = x+4 \longrightarrow x = u-4$

$du = dx \qquad 2x+1 = 2(u-4)+1$

$= 2u-7$

$= \int\left(2u^{1/2} - 7u^{-1/2}\right)du$

$= \frac{4}{3}u^{3/2} - 14u^{1/2} + C$

$= \boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$