Integral - Ch 5 - Substitution with Exponenial, Log, and Inverse Trig Functions 07dec2015

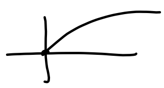
4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd
Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106

Test 2 - next week
Tues?

4.4 find c guanteed by MVT for Integrals

#45 $f(x) = x - 2\sqrt{x}$, $[0,2]$

continuous on $[0,2]$ ✓



$$f(c)\cdot(\underbrace{2-0}_{=2}) = \int_0^2 (x - 2\sqrt{x})\,dx$$

$$c - 2\sqrt{c} = \frac{1}{2}\int_0^2 (x^1 - 2x^{1/2})\,dx$$

$$c - 2\sqrt{c} = \frac{1}{2}\left[\frac{1}{2}x^2 - \frac{4}{3}x^{3/2}\right]\Big|_0^2$$

$$c - 2\sqrt{c} = \frac{1}{2}\left[\frac{1}{2}(2)^2 - \frac{4}{3}(2)^{3/2}\right]$$

$$c - 2\sqrt{c} = \frac{1}{2}\left[2 - \frac{8}{3}\sqrt{2}\right] = 1 - \frac{4\sqrt{2}}{3}$$

$$c - 2\sqrt{c} = 1 - \frac{4\sqrt{2}}{3}$$

Let $u = \sqrt{c}$

$u^2 = c$

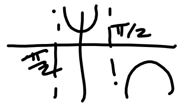
$$u^2 - 2u + \left(-1 + \frac{4\sqrt{2}}{3}\right) = 0$$

$$u = \frac{2 \pm \sqrt{(-2)^2 - 4(1)\left(-1 + \frac{4\sqrt{2}}{3}\right)}}{2(1)} = \sqrt{c}$$

$$\left(\qquad\qquad\right)^2 = c$$

pick $c \in [0,2]$

4.4 #47 $\quad (b-a)f(c) = \int_a^b f(x)\,dx$



$f(x) = 2\sec^2 x$, $[-\pi/4, \pi/4]$

continuous ✓

$$2\sec^2(c) = \frac{1}{\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)} = \int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx$$

$$= \frac{2}{\pi}\left[2\tan x\right]\Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{2}{\pi}\left[2\cdot 1 - 2(-1)\right]$$

$$2\sec^2(c) = \frac{2}{\pi}\cdot 4$$

$$\sec^2(c) = \frac{4}{\pi}$$

$$\sec(c) = \pm\frac{2}{\sqrt{\pi}}$$

$$c = \sec^{-1}\left(\frac{2}{\sqrt{\pi}}\right), \sec^{-1}\left(-\frac{2}{\sqrt{\pi}}\right)$$

pick $c \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

104. $\displaystyle\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

$$= \int \frac{e^{2x}}{e^x}\,dx + \int \frac{2e^x}{e^x}\,dx + \int \frac{1}{e^x}\,dx$$

$$= \int e^x\,dx + \int 2\,dx + \int e^{-x}\,dx$$

$$= e^x + 2x + \int e^{-x}\,dx \quad + c$$

Let $u = -x$

$du = -dx$

$-du = dx$

$$= e^x + 2x + \int -e^u\,du \quad + c$$

$$= e^x + 2x - e^u + c$$

$$= \boxed{e^x + 2x - e^{-x} + c}$$

108. $\int \ln(e^{2x-1})dx$

$\frac{d}{dx}\ln x = \frac{1}{x}$

$\int \frac{1}{x}dx = \ln|x| + c$

$= \int (2x-1)dx$

$= x^2 - x + C$

5.5

68. $\int 2^{\sin x}\cos x\,dx = \int 2^u du$

$u = \sin x$

$du = \cos x\,dx$

$= 2^u \cdot \frac{1}{\ln 2} + c$

$= 2^{\sin x} \cdot \frac{1}{\ln 2} + c$

64. $\int_{-2}^{0} (3^3 - 5^2)\, dx$

$= \int_{-2}^{0} 2\, dx = 2x \Big|_{-2}^{0} = 2(0) - 2(-2) = \boxed{4}$

5.4

102. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\, dx = \int \frac{2(e^x - e^{-x})}{(e^x + e^{-x})^2}\, dx$

$u = e^x + e^{-x}$

$du = (e^x - e^{-x})\, dx$

$$\boxed{\begin{aligned}(e^{-x})' &= e^{-x} \cdot (-x)' \\ &= -e^{-x}\end{aligned}}$$

$= \int \frac{2\, du}{u^2} = \int 2u^{-2}\, du$

$= -2u^{-1} + C$

$= -\frac{2}{u} + C$

$= \boxed{-\frac{2}{e^x + e^{-x}} + C}$

5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$\arcsin x = \sin^{-1}(x)$

$\sin(\pi/6) = 1/2$

$\arcsin(1/2) = \pi/6$

2. $$\int \frac{3dx}{\sqrt{1-4x^2}} = \int \frac{3dx}{\sqrt{1-(2x)^2}}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$u = 2x$; $a = 1$

$\frac{3}{2} du = 2dx \cdot \frac{3}{2}$

$\frac{3}{2} du = 3dx$

$$= \int \frac{3}{2} \cdot \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{3}{2} \arcsin\frac{2x}{1} + C$$

$$= \boxed{\frac{3}{2}\arcsin(2x) + C}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

8. $\displaystyle\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx$

$u = x$
$a = 3$

$$= \int_{\sqrt{3}}^{3} \frac{1}{3^2+x^2}\,dx = \frac{1}{3}\arctan\frac{x}{3}\Big|_{\sqrt{3}}^{3}$$

$$= \frac{1}{3}\arctan\frac{3}{3} - \frac{1}{3}\arctan\frac{\sqrt{3}}{3}$$

$$= \frac{1}{3}\arctan 1 - \frac{1}{3}\arctan\frac{1}{\sqrt{3}}$$

$$= \frac{1}{3}\cdot\pi/4 - \frac{1}{3}\cdot\pi/6$$

$$= \frac{\pi}{12} - \frac{\pi}{18}$$

$$= \frac{3\pi - 2\pi}{36} = \boxed{\frac{\pi}{36}}$$