4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd
Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106

Test #2 - Tues, 15 Dec.

4.5

#57 $\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du$

Let $u = x+2 \Rightarrow x = u-2$

$du = dx$

$= \int (u^1-2)u^{1/2}\,du$

$= \int \left(u^{3/2} - 2u^{1/2}\right)du$

#63 $\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx = \int \frac{-(u-1)}{u-\sqrt{u}}\,du =$

$u = x+1$
$du = dx$
$x = u-1$

$$= \int \frac{-(u-1)}{u-\sqrt{u}} \cdot \frac{u+\sqrt{u}}{u+\sqrt{u}}\,du =$$

$$= \int \frac{-(u-1)(u+\sqrt{u})}{u^2-u}\,du$$

$$= \int \frac{-\cancel{(u-1)}(u+\sqrt{u})}{u\cancel{(u-1)}}\,du$$

$$= \int \frac{-(u+\sqrt{u})}{u}\,du$$

$$= \int (-1-u^{-1/2})\,du$$

$$= -u - 2u^{1/2} + C$$

$$= -(x+1) - 2(x+1)^{1/2} + C$$

$$= \boxed{-(x+1) - 2\sqrt{x+1} + C}$$

5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$\arcsin x = \sin^{-1}(x)$

$\sin(\pi/6) = 1/2$

$\arcsin(1/2) = \pi/6$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+c$$

12. $\displaystyle\int \frac{x^4-1}{x^2+1}\,dx$

$$= \int \frac{(x^2-1)\cancel{(x^2+1)}}{\cancel{x^2+1}}\,dx$$

$$= \int (x^2-1)\,dx = \boxed{\tfrac{1}{3}x^3 - x + C}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+c$$

16. $\displaystyle\int \frac{1}{x\sqrt{x^4-4}}\,dx$

$$= \int \frac{1}{x\sqrt{(x^2)^2-2^2}}\,dx$$

$$= \int \frac{1}{x\sqrt{(x^2)^2-2^2}}\cdot\frac{x}{x}\,dx = \int \frac{x\,dx}{x^2\sqrt{(x^2)^2-2^2}}$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int \frac{1}{2}\cdot\frac{du}{u\sqrt{u^2-2^2}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}\operatorname{arcsec}\frac{x^2}{2} + C$$

$$= \boxed{\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

30. $\int \frac{x-2}{(x+1)^2+4}\,dx$

$u-3 = x-2$

$u = x+1$

$du = dx$

$$= \int \frac{u-3}{u^2+2^2}\,du$$

$$= \int \frac{u\,du}{u^2+2^2} - \int \frac{3\,du}{u^2+2^2}$$

$$= \int \frac{u\,du}{u^2+4} - \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$v = u^2+4$

$dv = 2u\,du$

$\frac{1}{2}dv = u\,du$

$$= \int \frac{1}{2}\cdot\frac{dv}{v} - \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$$= \frac{1}{2}\ln|v| - \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$$= \frac{1}{2}\ln(u^2+4) - \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$$= \boxed{\frac{1}{2}\ln\left[(x+1)^2+4\right] - \frac{3}{2}\arctan\frac{x+1}{2} + C}$$

32. $\int_{-2}^{2} \frac{dx}{x^2+4x+13}$

$$\int \frac{du}{a^2+u^2}$$

?

Completing the square:

$ax^2+bx+c \sim A(x-h)^2+K$

$a\left(\frac{b}{2a}\right)^2$

$(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$a(x^2+\frac{b}{a}x)+c$

$a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right)+c-\frac{b^2}{4a}$

$a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$

32. $\displaystyle\int_{-2}^{2}\frac{dx}{x^2+4x+13}=\int_{-2}^{2}\frac{dx}{(x+2)^2+3^2}$

$x^2+4x+13$

$x^2+4x+4+13-4$

$(x+2)^2+9$

$=\frac{1}{3}\arctan\frac{x+2}{3}\Big|_{-2}^{2}$

$=\frac{1}{3}\arctan\frac{4}{3}-\frac{1}{3}\arctan 0$

$=\boxed{\frac{1}{3}\arctan\frac{4}{3}}$