Test #2 - Tuesday, 15 December

(TOMORROW!)

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2}$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= \boxed{-\arctan(\cos x) + C}$$

$$\int \frac{e^{4x} - e^{2x} + 1}{e^{x}}\,dx$$

$$= \int (e^{3x} - e^{x} + e^{-x})\,dx$$

$$= \int e^{3x}\,dx - \int e^{x}\,dx + \int e^{-x}\,dx$$

$u = 3x$
$du = 3dx$

$u = -x$
$du = -dx$

$$= \boxed{\frac{1}{3}e^{3x} - e^{x} - e^{-x} + C}$$

$$\int_0^3 f(x)\,dx = 4 \quad , \quad \int_3^6 f(x)\,dx = -1$$

$$\int_0^6 = \int_0^3 + \int_3^6 \qquad \int_6^3 = -\int_3^6$$

$$\int_4^4 = 0 \qquad \int_3^6 10f(x)\,dx = 10\int_3^6 f(x)\,dx$$

$$2\pi\int_0^1 (y+1)\sqrt{1-y}\,dy = 2\pi\int_{y=0}^{y=1} -(2-u)\underset{u^{1/2}}{\underset{\shortparallel}{\sqrt{u}}}\,du$$

$u = 1-y \rightsquigarrow y-1 = -u$

$\quad\quad\quad\quad\quad +2 \quad +2$

$du = -dy \quad\quad y+1 = -u+2$

$-du = dy \quad\quad\quad = 2-u$

$$\to 2\pi\int_{y=0}^{1}\left(u^{3/2} - 2u^{1/2}\right)du$$

$$= 2\pi\left[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}\right]_{y=0}^{1}$$

$$= 2\pi\left[\frac{2}{5}(1-y)^{5/2} - \frac{4}{3}(1-y)^{3/2}\right]_0^1$$

$$= -2\pi\left[\frac{2}{5} - \frac{4}{3}\right]$$

$$= -2\pi\cdot\frac{2}{5}\cdot\frac{3}{3} + 2\pi\cdot\frac{4}{3}\cdot\frac{5}{5}$$

$$= \frac{-12\pi}{15} + \frac{40\pi}{15} = \boxed{\frac{28\pi}{15}}$$

$$\int\frac{1}{e^{2x}+e^{-2x}}\,dx = \int\frac{1}{e^{2x}+e^{-2x}}\cdot\frac{e^{2x}}{e^{2x}}\,dx$$

$$= \int\frac{e^{2x}}{e^{4x}+1}\,dx = \int\frac{e^{2x}\,dx}{(e^{2x})^2+1^2}$$

$u = e^{2x}$

$du = e^{2x}\cdot 2\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

$$= \int\frac{\frac{1}{2}du}{u^2+1}$$

$$= \boxed{\frac{1}{2}\arctan\left(e^{2x}\right)+C}$$

$$\int (1+\sec \pi x)^2 \sec \pi x \tan \pi x \, dx$$

$u = 1 + \sec \pi x$

$du = \sec \pi x \tan \pi x \cdot \pi \, dx$

$\frac{1}{\pi} du = \sim$

$$\int \frac{1}{\pi} u^2 \, du$$

$$\int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln|\ln x| + C$$

$u = \ln x$

$du = \frac{dx}{x}$

52. $\displaystyle\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx = \int \frac{1/2\,du}{u}$

$= \frac{1}{2}\ln\left(e^{2x}+e^{-2x}\right)+C$

$u = e^{2x}+e^{-2x}$

$du = \left(2e^{2x}-2e^{-2x}\right)dx$

$= 2\left(e^{2x}-e^{-2x}\right)dx$

$\displaystyle\int \frac{\arctan(x/2)}{4+x^2}\,dx = \int \frac{1}{\cancel{4}\,2}u\,du = \frac{1}{\cancel{8}\,4}u^2+C$

$= \boxed{\frac{1}{\cancel{8}\,4}\left(\arctan\frac{x}{2}\right)^2+C}$

$u = \arctan\frac{x}{2}$

$du = \frac{1}{1+\left(\frac{x}{2}\right)^2}dx \cdot \frac{1}{2}$

$= \frac{1}{1+\frac{x^2}{4}}dx \cdot \frac{1}{2}\cdot\frac{4}{4} = \frac{4\,dx}{4+x^2} = \frac{du}{\cancel{4}\,2}$

$$(x^2+1)^3 = (x^2)^3 + 3(x^2)^2 \cdot 1 + 3x^2(1)^2 + 1^3$$

$$\int \tan^n x \sec^2 x \, dx \quad , n \neq -1$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c$$

$$\boxed{\frac{1}{n+1} \tan^{n+1} + C}$$

$n = -1$

$$\int u^n \, du = \int \frac{du}{u} = \ln|u| + c$$

$$\int [2 + f(x)]\,dx$$

$$\int_a^b 2\,dx + \int_a^b f(x)\,dx$$

$$2x \Big|_a^b$$

$\int \cot^4 x \csc^2 x\,dx$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$\int -u^4\,du$

$-\frac{1}{5}u^5 = -\frac{1}{5}\cot^5 x + C$

Alex S.

50) $\int \frac{e^{1/x}}{x^2}\,dx = \int -e^u\,du$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2} = -\frac{1}{x^2}\,dx$

$-du = \frac{1}{x^2}$

$-e^u \Rightarrow \boxed{-e^{1/x} + C}$

Kendall

1. $f(x)=4-x^2, [0,2]$

$f(c)\cdot(b-a)=\int_a^b f(x)\,dx$

$(4-c^2)\cdot(2-0)=\int_0^2 (4-x^2)\,dx$

$(4-c^2)\cdot 2 = 4x-\frac{1}{3}x^3\Big|_0^2$

$8-2c^2 = 4(2)-\frac{1}{3}(2)^3-0$

$\cancel{8}-2c^2=\cancel{8}-\frac{8}{3}$

$\sqrt{c^2}=\sqrt{\frac{4}{3}}$ $\quad c=\sqrt{\frac{4}{3}}$

Lauren

$\int_0^{\pi/4} \tan\left(\frac{\pi}{4}-x\right)dx = \int_0^{\pi/4} -\tan(u)\,du$

$u=\pi/4 - x$

$du=-dx$

$-du=dx$

$=-\ln|\cos u|\Big|_{x=0}^{x=\pi/4}$

$=-\ln|\cos(\pi/4-x)|\Big|_0^{\pi/4}$

$=(-\ln|\cos(\pi/4-\pi/4)|)$
$-(-\ln|\cos(\pi/4-0$

$=\boxed{\ln\left(\frac{\sqrt{2}}{2}\right)}$

Jameson