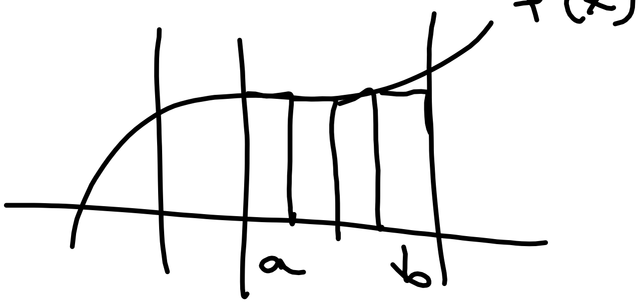
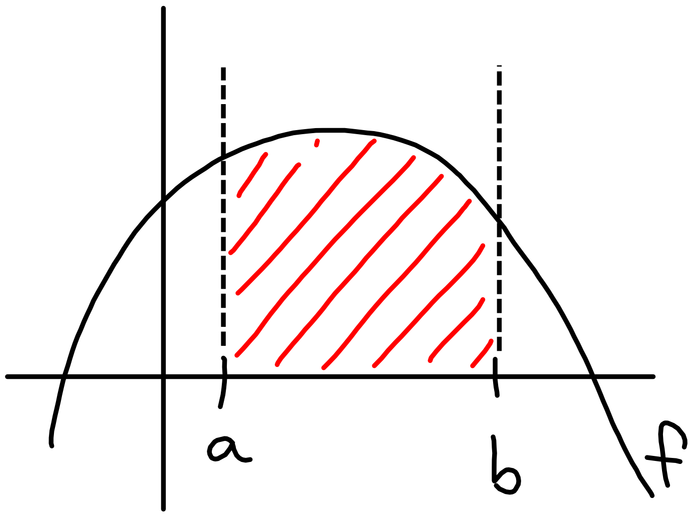
$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(a+i\cdot\frac{b-a}{n}\right)\cdot\underbrace{\frac{b-a}{n}}_{\text{width of each rectangle}}$$

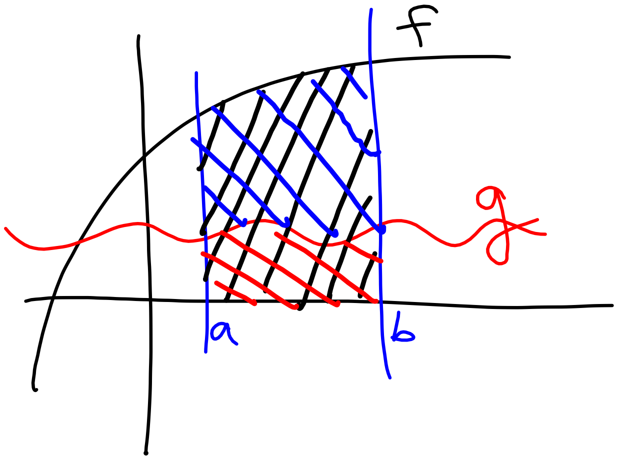


6.1 - Area Between Curves



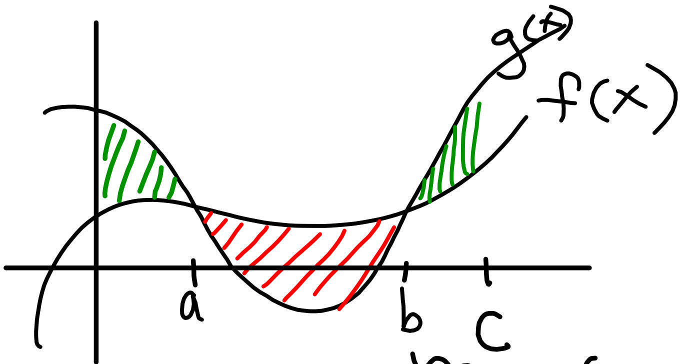
Area of region bounded by f(x) & x-axis, between a and b is

$$\int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

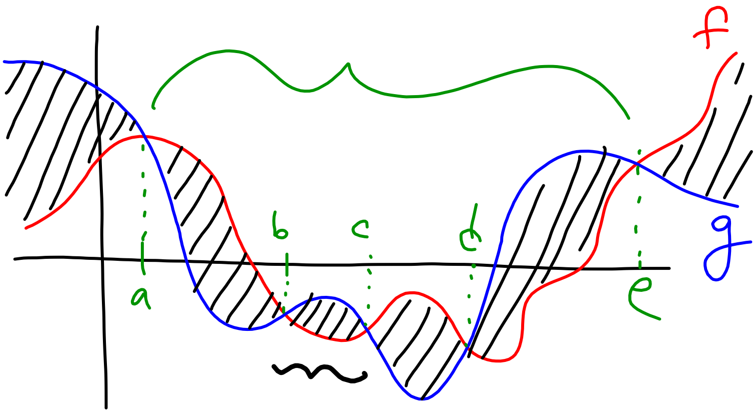
$$= \int_a^b [f(x) - g(x)]\,dx$$



$$\int_b^c [g(x) - f(x)]\,dx$$

$$\int_0^a [g(x) - f(x)]\,dx$$

$$\int_a^b [f(x) - g(x)]\,dx$$



$$-\int_b^c f(x)\,dx - \left(-\int_b^c g(x)\,dx\right) = \int_b^c [g(x) - f(x)]\,dx$$

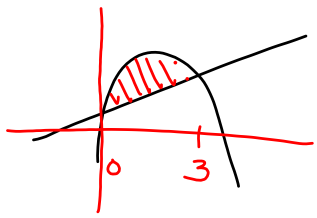
$$\int_a^b [f(x) - g(x)]\,dx + \int_b^c [g(x) - f(x)]\,dx + \int_c^d [f(x) - g(x)]\,dx$$

$$+ \int_d^e [g(x) - f(x)]\,dx$$

6.1 Find the area between the curves.

#18. $f(x) = -x^2 + 4x + 1$

$g(x) = x + 1$



$-x^2 + 4x + 1 = x + 1$

$0 = x^2 - 3x$

$0 = x(x-3) \qquad x = 0, 3$

$$\int_0^3 [(-x^2 + 4x + 1) - (x+1)]\,dx$$

$$= \int_0^3 (-x^2 + 3x)\,dx = -\tfrac{1}{3}x^3 + \tfrac{3}{2}x^2 \Big|_0^3$$

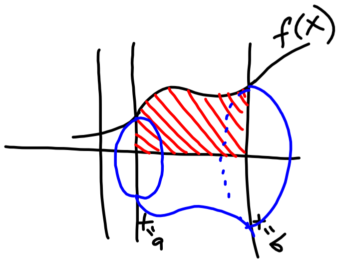
$$= \left[-\tfrac{1}{3}(3)^3 + \tfrac{3}{2}(3)^2\right] - \left[-\tfrac{1}{3}(0)^3 + \tfrac{3}{2}(0)^2\right] =$$

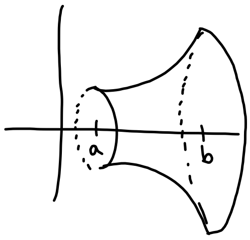
$$= -\frac{1}{3}\cdot\frac{27}{1} + \frac{3}{2}\cdot 9 =$$

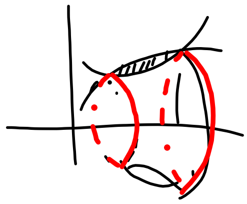
$$= -\frac{9}{1}\cdot\frac{2}{2} + \frac{27}{2}$$

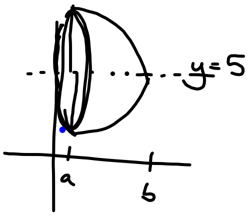
$$= -\frac{18}{2} + \frac{27}{2} = \boxed{\frac{9}{2}}$$

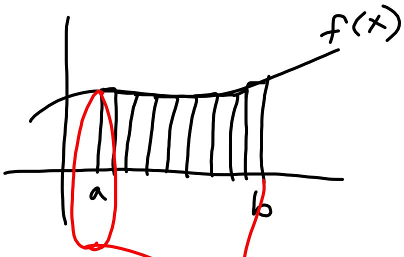
6.2 Volume of Solids of Revolution

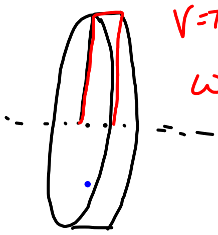












$V = \pi r^2 \cdot \omega$

$\omega = \Delta x$
$= dx$

$$\int_a^b \pi (f(x))^2 \cdot dx$$

$$\int_a^b \pi r^2 \cdot dx$$

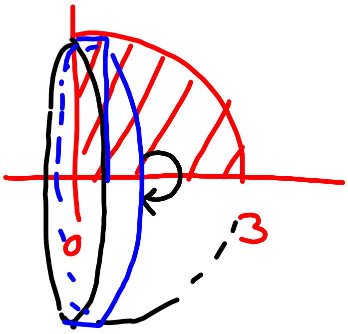
$r =$ some function of f

6.2

Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

4. $y=\sqrt{9-x^2}$

$y=\sqrt{a^2-x^2}$
top half of circle of radius a



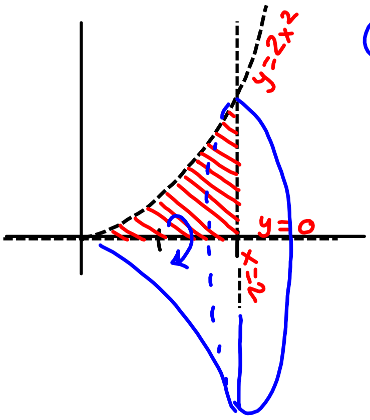
$$\int_0^3 \pi\left(\sqrt{9-x^2}\right)^2 \cdot dx$$
$$=\pi\int_0^3 (9-x^2)\,dx$$
$$=\pi\left[9x-\tfrac{1}{3}x^3\right]\Big|_0^3$$
$$=\pi\left[9\cdot3-\tfrac{1}{3}\cdot27\right]$$
$$=\pi[27-9]=\boxed{18\pi}$$

$V_{hemisphere}=$
$\frac{1}{2}\cdot\frac{4}{3}\pi r^3$
$=\frac{2}{3}\pi(3)^3$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations:

12. $y=2x^2$, $y=0$, $x=2$

about the line: (a) y-axis, (b) x-axis, (c) $y=8$, (d) $x=2$



(b) x-axis

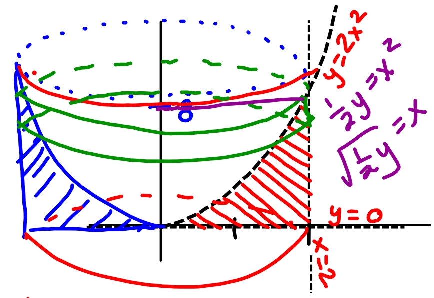
$$\int_0^2 \pi\left(2x^2\right)^2 dx=\int_0^2 4\pi x^4\,dx$$
$$=\frac{4\pi}{5}x^5\Big|_0^2=\frac{4\pi}{5}\cdot32=\boxed{\frac{128\pi}{5}}$$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations:

12. $y = 2x^2$, $y = 0$, $x = 2$

(a) y-axis

$x = \sqrt{y/2}$



Volume of solid = volume of outer cylinder minus volume of "inside of bowl"

$V_{\text{outer cylinder}} = \pi r^2 h = \pi(2)^2 \cdot 8 = 32\pi$

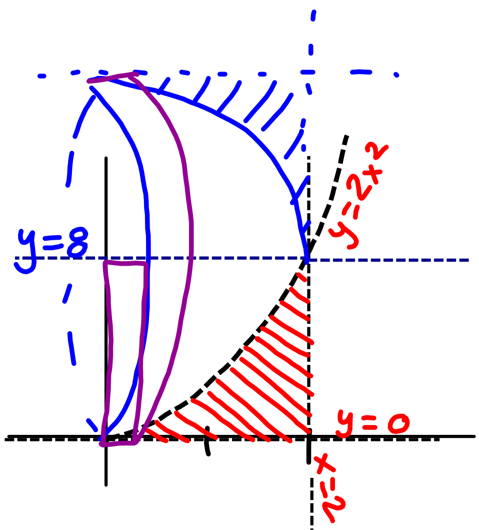
$V_{bowl} = \int_0^8 \pi\left(\sqrt{\tfrac{1}{2}y}\right)^2 dy = \int_0^8 \frac{\pi}{2} y\, dy = \frac{\pi}{4} y^2 \Big|_0^8$

$= 16\pi$

$V_{solid} = 32\pi - 16\pi$

$= \boxed{16\pi}$

(c) $y = 8$



$V_{\text{outer cylinder}} = \pi r^2 h$

$= \pi(8)^2 \cdot 2 = 128\pi$

$V_{\text{inner bowl}} = \int_0^2 \pi\left(8 - 2x^2\right)^2 dx$

Homework:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35