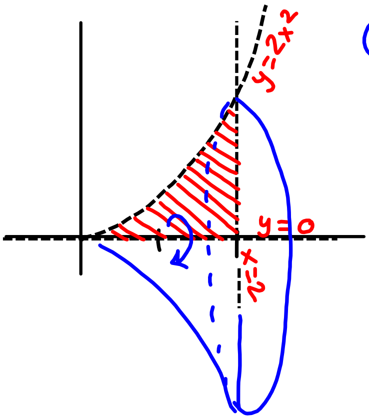
Find the volume of the solid generated by revolving the region bounded by the graphs of the equations:

12. $y = 2x^2$, $y = 0$, $x = 2$

about the line: (a) y-axis, (b) x-axis, (c) $y = 8$, (d) $x = 2$



(b) x-axis

$$\int_0^2 \pi \left(2x^2\right)^2 dx = \int_0^2 4\pi x^4 \, dx$$

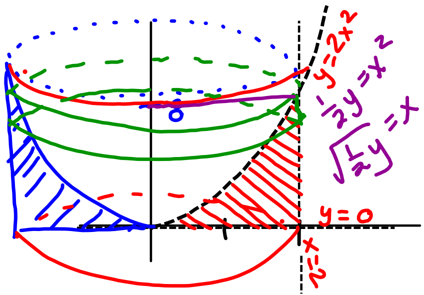
$$= \frac{4\pi}{5} x^5 \Big|_0^2 = \frac{4\pi}{5} \cdot 32 = \boxed{\frac{128\pi}{5}}$$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations:

12. $y = 2x^2$, $y = 0$, $x = 2$

(a) y-axis

$x = \sqrt{y/2}$



Volume of solid = volume of outer cylinder minus volume of "inside of bowl"

$$V_{\substack{\text{outer}\\\text{cylinder}}} = \pi r^2 h = \pi (2)^2 \cdot 8 = 32\pi$$

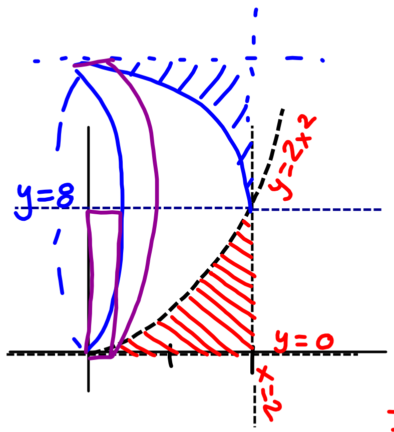
$$V_{\text{bowl}} = \int_0^8 \pi \left(\sqrt{\tfrac{1}{2}y}\right)^2 dy = \int_0^8 \frac{\pi}{2} y \, dy = \frac{\pi}{4} y^2 \Big|_0^8$$

$$= 16\pi$$

$$V_{\text{solid}} = 32\pi - 16\pi$$

$$= \boxed{16\pi}$$

(c) $y=8$



$$V_{\text{outer cylinder}} = \pi r^2 h$$

$$= \pi (8)^2 \cdot 2 = 128\pi$$

$$V_{\text{inner bowl}} = \int_0^2 \pi \left(8 - 2x^2\right)^2 dx$$

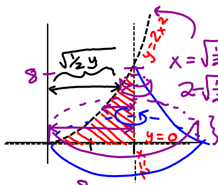
$$= \int_0^2 \pi \left(64 - 32x^2 + 4x^4\right) dx$$

$$= \pi \left(64x - \frac{32}{3}x^3 + \frac{4}{5}x^5\right)\Big|_0^2$$

$$= \pi \left(64 \cdot 2 - \frac{32}{3} \cdot 8 + \frac{4}{5} \cdot 32\right) = \frac{1024}{15}\pi$$

$$V_{\text{total}} = 128\pi - \frac{1024}{15}\pi = \boxed{\frac{896}{15}\pi}$$

(d) $x=2$



$x = \sqrt{\tfrac{1}{2}y}$

$2 - \sqrt{\tfrac{1}{2}y} = r$

$dy = h$

$$V = \int_0^8 \pi \left(2 - \sqrt{\tfrac{1}{2}y}\right)^2 dy$$

$$= \int_0^8 \pi \left(4 - 4\sqrt{\tfrac{1}{2}y} + \tfrac{1}{2}y\right) dy$$

$$= \int_0^8 \left(4\pi - \frac{4\pi}{\sqrt{2}} y^{1/2} + \frac{\pi}{2} y\right) dy$$

$$= 4\pi y - \frac{8\pi}{3\sqrt{2}} y^{3/2} + \frac{\pi}{4} y^2 \Big|_0^8$$

$$= 32\pi - \frac{8\pi}{3\sqrt{2}}(8)^{3/2} + 16\pi$$

$$= 48\pi - \frac{8\pi}{3\sqrt{2}}(8)^{3/2}$$

$$= 48\pi - \frac{8\pi}{3\sqrt{2}}\left(\sqrt{8}\right)^3$$

$$(2\sqrt{2})^3$$

$$= 48\pi - \frac{8\pi}{3\cancel{\sqrt{2}}} \cdot 8 \cdot 2\cancel{\sqrt{2}}$$

$$= 48\pi - \frac{128\pi}{3}$$

$$= \boxed{\frac{16\pi}{3}}$$

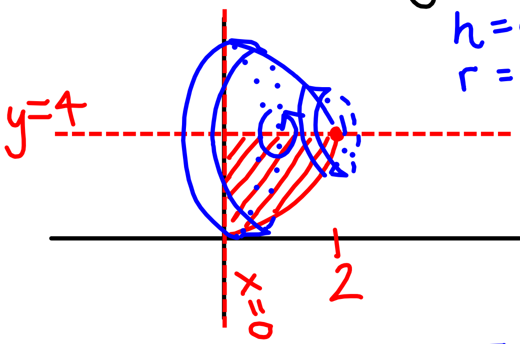
16. $y = \frac{1}{2}x^3$, $y = 4$, $x = 0$

rotate about $y = 4$

$4 = \frac{1}{2}x^3$

$8 = x^3$

$2 = x$



$h = dx$

$r = 4 - \frac{1}{2}x^3$

$$V = \int_0^2 \pi\left(4 - \frac{1}{2}x^3\right)^2 dx$$

$$= \int_0^2 \pi\left(16 - 4x^3 + \frac{1}{4}x^6\right) dx$$

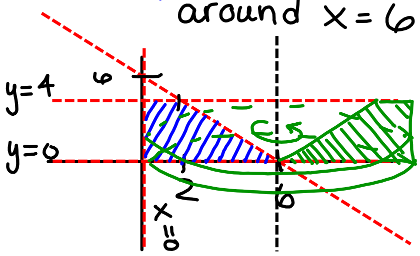
$$= 16\pi x - \pi x^4 + \frac{\pi}{28}x^7 \Big|_0^2$$

$$= 16\pi(2) - \pi(16) + \frac{\pi}{28}(2)^7 = \boxed{\frac{144\pi}{7}}$$

6.2

20. $y = 6 - x$, $y = 0$, $y = 4$, $x = 0$ $\quad y = -1x + 6$ $\quad x = 6 - y$

around $x = 6$



$V_{\substack{outer \\ cylinder}} = \pi(6)^2(4)$

$= 144\pi$

$V_{\substack{inner \\ cone}} = \int_a^b \pi r^2 h$

$= \int_0^4 \pi(6 - (6 - y))^2 dy$

$= \int_0^4 \pi y^2 dy$

$= \frac{\pi}{3}y^3 \Big|_0^4$

$= \frac{64\pi}{3}$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(4)^2 \cdot 4$

$= \frac{64\pi}{3}$

$V_{total} = 144\pi - \frac{64\pi}{3}$

$= \boxed{\frac{368\pi}{3}}$

Homework:

- 6.1 #1-9 odd; 19, 43 — area between curves
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 — volume of solids of revolution
- 6.4 #5, 7, 13, 33, 35 — surface area of solids of revolution