Homework:

- 6.1 #1-9 odd; 19, 43 — area between curves
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 — volume of solids of revolution
- 6.4 #5, 7, 13, 33, 35 — arc length & surface area of solids of revolution

- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts

Test #3 soon (like next week)? Fri 01/15

Add a 4th test?

$f(x) = 3(x^3 - x)$

$g(x) = 0$

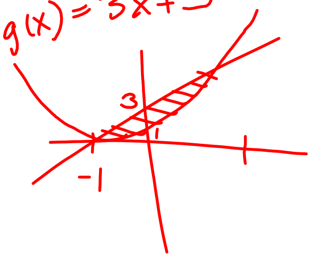
$$A = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} -f(x)\,dx$$

$$= -2\int_{0}^{1} f(x)\,dx$$

$$= 2\int_{-1}^{0} f(x)\,dx$$

$f(x)=x^2+2x+1=(x+1)^2$

$g(x)=3x+3$



$x^2+2x+1=3x+3$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=-1,2$

$$\int_{-1}^{2}\left[(3x+3)-(x^2+2x+1)\right]dx$$

$$=\int_{-1}^{2}(-x^2+x+2)\,dx$$

$$=-\frac{1}{3}x^3+\frac{1}{2}x^2+2x\Big|_{-1}^{2}$$

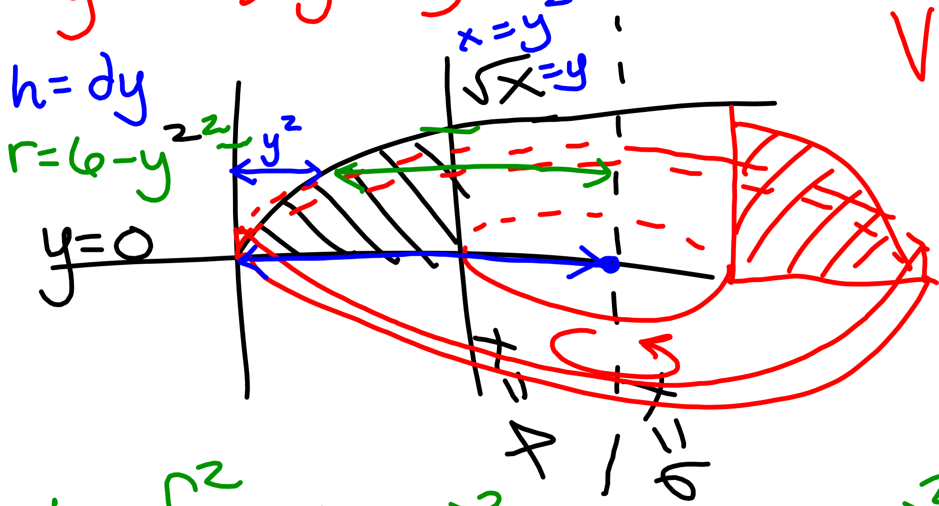
$$=\left(\frac{-8}{3}+2+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$$

$$=\frac{-9}{3}-\frac{1}{2}+8$$

$$=5-\frac{1}{2}$$

$$=4.5$$

$y=\sqrt{x},\ y=0,\ x=4$ about $x=6$

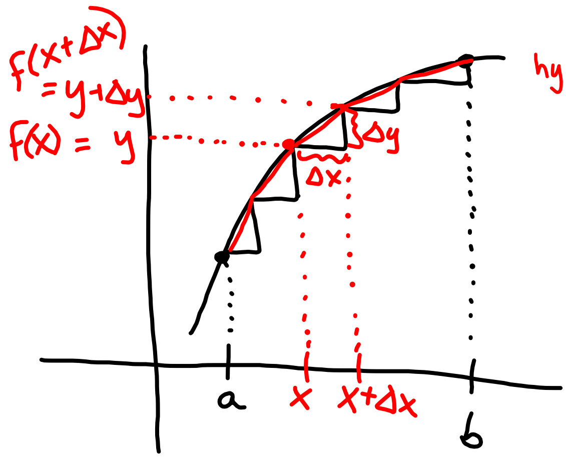


V_{total} = outer integral − inner cylinder

$$V=\int_0^2\pi(6-y^2)^2\,dy-\pi(2)^2\cdot 2$$

6.4 - Arc Length & Surfaces of Revolution

The arc length s of a smooth curve f from a to b is
$$s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$



hypotenuse has length $\sqrt{(\Delta x)^2+(\Delta y)^2}$
$$= \sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$$
$$= \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot \Delta x$$

MVT $\quad f(x+\Delta x)-f(x) = f'(c)\cdot \Delta x$
$$\frac{\Delta y}{\Delta x} = f'(c)$$

6. $y = \frac{3}{2}x^{2/3}+4, \quad [1,27]$

$$S = \int_a^b \sqrt{1+[f'(x)]^2}\,dx = \int_1^{27}\sqrt{1+\left(\frac{1}{\sqrt[3]{x}}\right)^2}\,dx$$

$$y' = x^{-1/3} = \frac{1}{\sqrt[3]{x}}$$

$$= \int_1^{27}\sqrt{\left(\frac{1}{\sqrt[3]{x}}\right)^2\left(\left(\sqrt[3]{x}\right)^2+1\right)}\,dx$$

$$= \int_1^{27}\frac{1}{\sqrt[3]{x}}\sqrt{\left(\sqrt[3]{x}\right)^2+1}\,dx$$

Let $u = x^{2/3}+1$

$du = \frac{2}{3}x^{-1/3}dx$

$\frac{3}{2}du = \frac{dx}{\sqrt[3]{x}}$

$$= \int_{x=1}^{x=27}\frac{3}{2}u^{1/2}\,du$$

$$= \frac{3}{2}\cdot\frac{2}{3}u^{3/2}\Big|_{x=1}^{27} = \left(x^{2/3}+1\right)^{3/2}\Big|_1^{27}$$

$$= \left(\left(\sqrt[3]{27}\right)^2+1\right)^{3/2} - \left(\left(\sqrt[3]{1}\right)^2+1\right)^{3/2}$$

$$= 10^{3/2} - 2^{3/2}$$

$$= \sqrt{1000} - \sqrt{8}$$

$$= \boxed{10\sqrt{10} - 2\sqrt{2}}$$

18. $y = \ln x$, $[1,5]$

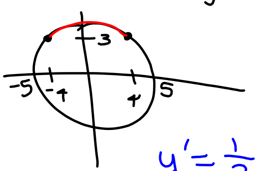
$y' = \frac{1}{x}$

$$S = \int_1^5 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx = \int_1^5 \sqrt{\frac{1}{x^2}(x^2+1)}\, dx$$

$$= \int_1^5 \frac{\sqrt{x^2+1}}{x}\, dx$$

We observe that IDK

32. Find arc length from $(-3,4)$ clockwise to $(4,3)$ along the circle $x^2+y^2=25$.



$y^2 = 25 - x^2$

top half $y = \sqrt{25-x^2}$
$= (25-x^2)^{1/2}$

$y' = \frac{1}{2}(25-x^2)^{-1/2} \cdot (-2x)$

$y' = \frac{-x}{\sqrt{25-x^2}}$

$$S = 2\int_0^4 \sqrt{1 + \left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\, dx$$

$$= 2\int_0^4 \sqrt{1 + \frac{x^2}{25-x^2}}\, dx = 2\int_0^4 \sqrt{\frac{25-x^2}{25-x^2} + \frac{x^2}{25-x^2}}\, dx$$

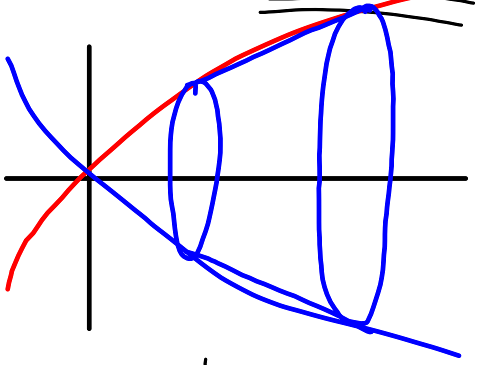
$$= 2\int_0^4 \sqrt{\frac{25}{25-x^2}}\, dx = 2\int_0^4 \frac{5}{\sqrt{25-x^2}}\, dx$$

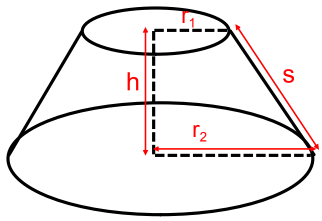
$$= 2 \cdot 5 \arcsin\frac{x}{5}\Big|_0^4$$

$$= 10 \arcsin\frac{4}{5} - 10 \arcsin 0$$

$$= \boxed{10 \arcsin 4/5}$$

Area of a Surface of Revolution





Truncated Cone:
$A = 2\pi \cdot r_{avg} \cdot s$

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx$$