Homework:

- 6.1 #1-9 odd; 19, 43 — area between curves
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 — volume of solids of revolution
- 6.4 #5, 7, 13, 33, 35 — arc length & surface area of solids of revolution

- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts

Test #3 - Friday, 15 Jan

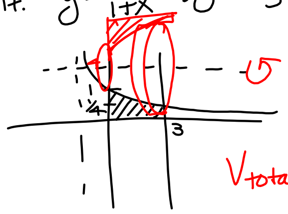
6.1, 6.2, 6.4, and review

No class tomorrow (Tues. 01/12)

Review Wednesday

6.2

17. $y=\frac{1}{1+x},\ y=0,\ x=0,\ x=3$



$V_{\text{outer cylinder}} = \pi(4)^2 \cdot 3$

$V_{\text{inner solid}} = \int_0^3 \pi\left(4-\frac{1}{1+x}\right)^2 dx$

$V_{total} = V_{cyl} - V_{solid}$

$$\pi\int_0^3 \left(16 - \frac{8}{1+x} - \frac{1}{(1+x)^2}\right)dx$$

$$= \left[\pi\cdot 16x\right]_0^3 - 8\pi\int_0^3 \frac{1}{1+x}\,dx - \pi\int_0^3 \frac{1}{(1+x)^2}\,dx$$

$u = 1+x$, $du = dx$ $\qquad$ $v = 1+x$, $dv = dx$

$$= 48\pi - 8\pi\int_{x=0}^3 \frac{du}{u} - \pi\int_{x=0}^3 \frac{1}{v^2}\,dv$$

$$= 48\pi - \left[8\pi \ln|1+x|\right]_0^3 + \left[\frac{\pi}{1+x}\right]_0^3$$

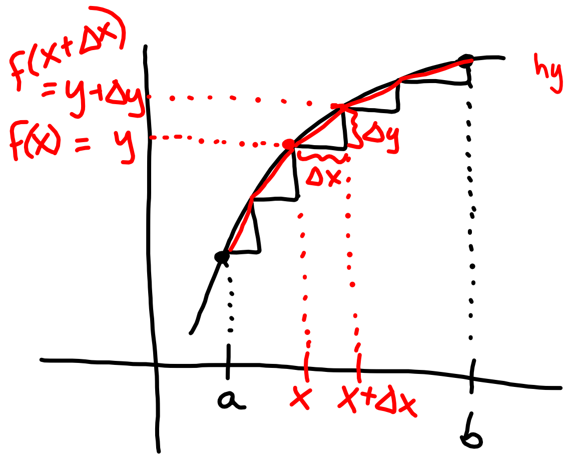
$$= 48\pi - 8\pi\ln 4 + \frac{\pi}{4} - \pi$$

$$V = 48\pi - \left[48\pi - 8\pi\ln 4 + \frac{\pi}{4} - \pi\right]$$

$$= 8\pi\ln 4 + \frac{3\pi}{4}$$

6.4 - Arc Length & Surfaces of Revolution

The arc length s of a smooth curve f from a to b is $\quad s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$



hypotenuse has length $\sqrt{(\Delta x)^2+(\Delta y)^2}$

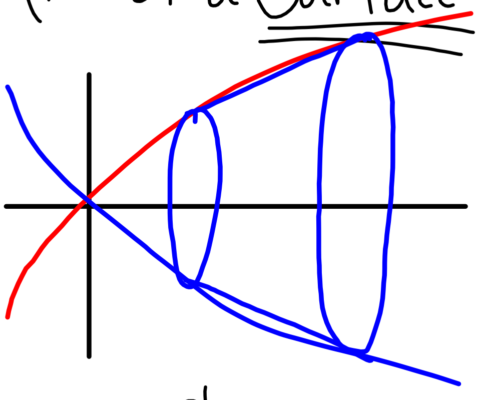
$= \sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$

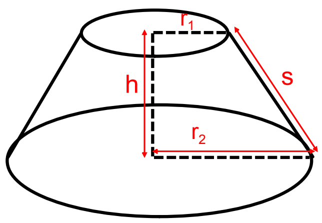
$= \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot \Delta x$

MVT $\quad f(x+\Delta x) - f(x) = f'(c)\cdot \Delta x$

$\frac{\Delta y}{\Delta x} = f'(c)$

Area of a Surface of Revolution



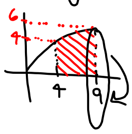


Truncated Cone:
$A = 2\pi \cdot r_{avg} \cdot s$

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S = 2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

34. $y = 2\sqrt{x}$, $[4,9]$ revolve about x-axis $r(x) = 2\sqrt{x}$; $f'(x) = \frac{1}{\sqrt{x}}$



$$\int_4^9 2\pi(2\sqrt{x})\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

$$= 4\pi\int_4^9 \sqrt{x}\sqrt{1+\frac{1}{x}}\,dx$$

$$= 4\pi\int_4^9 \sqrt{x+1}\,dx$$

$u = x+1$
$du = dx$

$$= 4\pi\int_{u=5}^{10} u^{1/2}\,du$$

$$= 4\pi\cdot\frac{2}{3}u^{3/2}\Big|_5^{10}$$

$$= \frac{8\pi}{3}\left(\sqrt{1000}-\sqrt{125}\right)$$

$$= \boxed{\frac{8\pi}{3}\left(10\sqrt{10}-5\sqrt{5}\right)}$$

7.1 Basic Integration Rules

$$\int\frac{4}{x^2+9}\,dx \qquad \int\frac{4x}{x^2+9}\,dx \qquad \int\frac{4x^2}{x^2+9}\,dx$$

$$= \boxed{\frac{4}{3}\arctan\frac{x}{3}+C}$$

$u = x^2+9$
$du = 2x\,dx$
$2du = 4x\,dx$

$$\int\frac{4x}{x^2+9}\,dx = 2\int\frac{du}{u} = \boxed{2\ln(x^2+9)+c}$$

$$\begin{array}{r} 4 \\ x^2+9\,\overline{\big)\,4x^2} \\ -(4x^2+36) \\ \hline -36 \end{array}$$

$$\int\frac{4x^2}{x^2+9}\,dx = \int\left(4-\frac{36}{x^2+9}\right)dx$$

$$= \boxed{4x-12\arctan\frac{x}{3}+c}$$

$$\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx$$

$$= \int \left(\frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}\right)dx$$

$$= \int \left(1 - \frac{e^x}{1+e^x}\right)dx = x - \int \frac{e^x\,dx}{1+e^x}$$

$u = 1+e^x$

$du = e^x\,dx$

$$= x - \int \frac{du}{u} = x - \ln|1+e^x| + C$$

$$= \boxed{x - \ln(1+e^x) + C}$$

$$\int \tan^2 2x\,dx$$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$$= \int (\sec^2 2x - 1)\,dx$$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$$= \boxed{\frac{1}{2}\tan 2x - x + C}$$

$$\int \cot x \underbrace{\ln(\sin x)}\, dx = \int u\, du = \frac{1}{2}u^2 + C$$

$$= \boxed{\frac{1}{2}(\ln(\sin x))^2 + C}$$

$u = \ln(\sin x)$

$du = \frac{1}{\sin x} \cdot \cos x\, dx$

$du = \cot x\, dx$