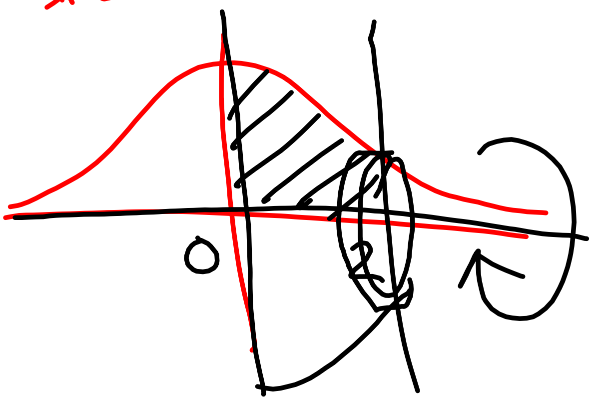
6.2 #35 $y=e^{-x^2}$, $y=0$, $x=0$, $x=2$

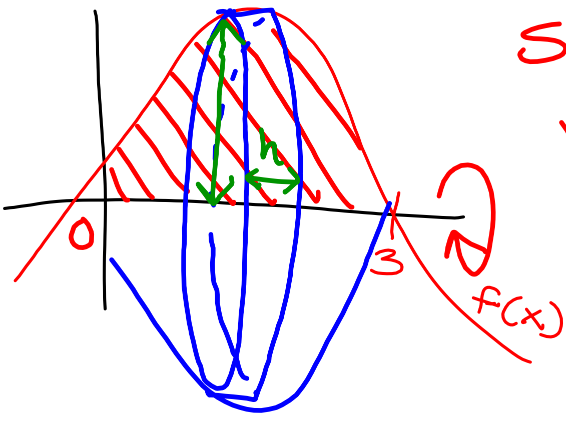


$h=dx$

$r=e^{-x^2}$

$$V=\int_0^2 \pi\left(e^{-x^2}\right)^2 dx$$

adjacent to x-axis, about x-axis

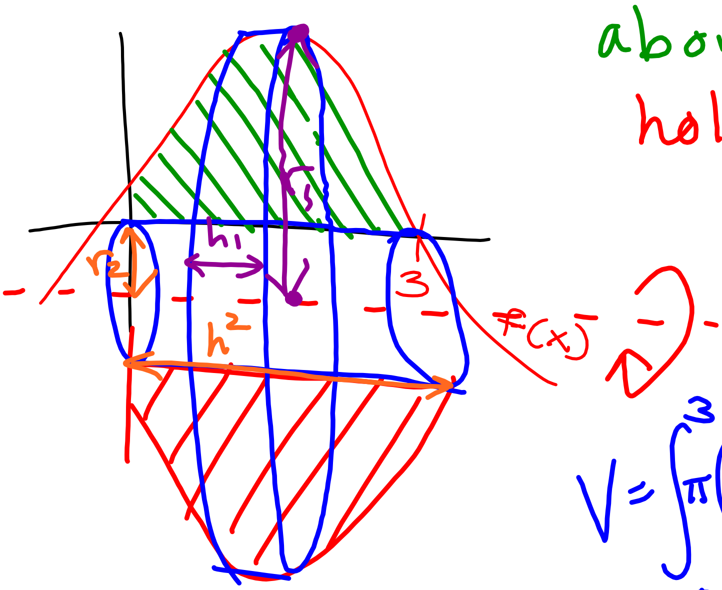


SOLID

$$V=\int_a^b \pi r^2 h$$

f(x) dx

$$=\int_0^3 \pi\left(f(x)\right)^2 dx$$



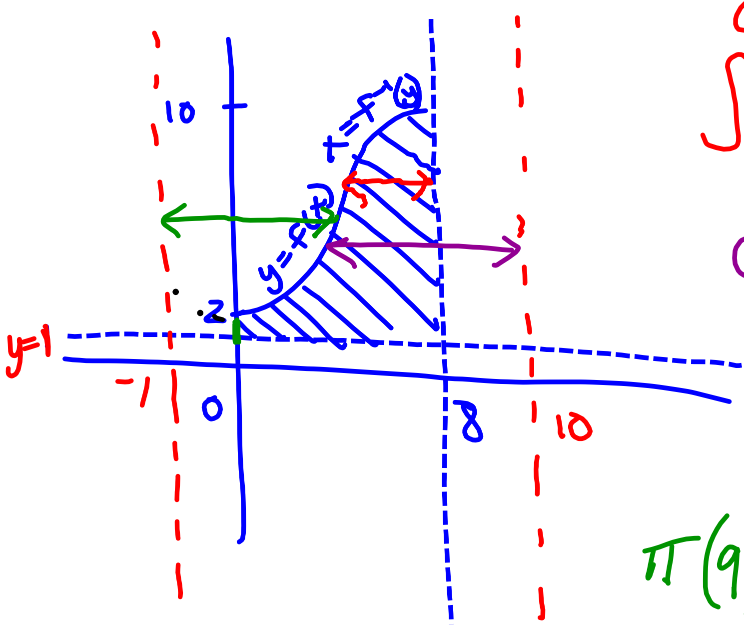
about $y=-2$

hollow

$$\int_a^b \pi r_1^2 h_1 - \pi r_2^2 h_2$$

$$V=\int_0^3 \pi(\ \ )^2\,dx - \pi(2)^2\cdot 3$$

$f(x)+2$



ⓐ $x=8$ solid

$$\int_1^{10} \pi\left(8-f^{-1}(y)\right)^2 dy$$

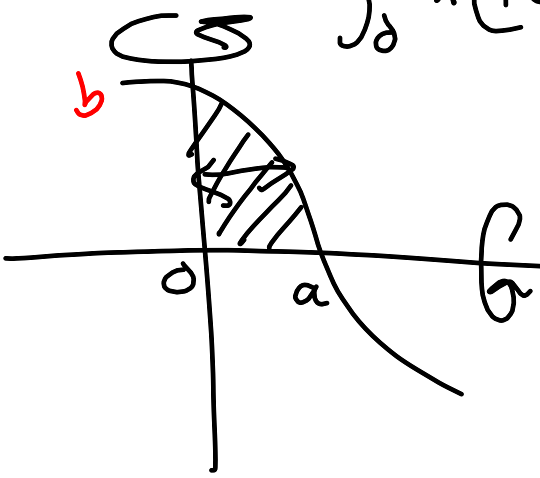
ⓑ $x=10$ hollow

$$\int_1^{10} \pi\left(10-f^{-1}(y)\right)^2 dy - \pi(2)^2\cdot 9$$

ⓒ $x=-1$ hollow

$$\pi(9)^2\cdot 9 - \left(\pi(1)^2\cdot 1 + \int_2^{10} \pi\left(f^{-1}(y)+1\right)^2 dy\right)$$

$$\int_0^b \pi [f(y)]^2 \, dy$$



$$\int_0^a \pi [f(x)]^2 \, dx$$

$$\int_1^x (t+5)\, dt = \frac{1}{2}t^2 + 5t \Big|_1^x$$

$$F(x) = \frac{1}{2}x^2 + 5x - \left(\frac{1}{2} + 5\right)$$

8. $F(x) = \int_{-2}^{x^3} \sin t^3 \, dt$

$F'(x) = \sin\left(x^3\right)^3 \cdot 3x^2$

$F(x) = \int_{p(x)}^{q(x)} f(t)\, dt$

$\int_a^b = \int_a^c + \int_c^b$

$\int_{p(x)}^{q(x)} = \int_{p(x)}^{c} + \int_c^{q(x)} = -\int_c^{p(x)} + \int_c^{q(x)}$

$F'(x) = -f(p(x)) \cdot p'(x) + f(q(x)) \cdot q'(x)$

13. $\int_0^{\pi/3} \underbrace{\sin x}\underbrace{\cos x\,dx} = \int_{x=0}^{x=\pi/3} u\,du = \frac{1}{2}u^2\Big|_{x=0}^{\pi/3}$

$= \frac{1}{2}(\sin x)^2\Big|_0^{\pi/3}$

$u = \sin x$

$du = \cos x\,dx$

$v = \cos x$

$dv = -\sin x\,dx$

$-dv = \sin x\,dx$

$x = 0$

$u = \sin 0 = 0$

$x = \pi/3$

$u = \sin \pi/3$

$= \sqrt{3}/2$

$= \int_{u=0}^{u=\sqrt{3}/2} u\,du = \frac{1}{2}u^2\Big|_0^{\sqrt{3}/2}$

$= \boxed{\frac{3}{8}}$

$\sin 2x = 2\sin x\cos x$

$\frac{1}{2}\sin 2x = \sin x\cos x$

$\int_a^b \frac{1}{2}\sin 2x\,dx = \int_a^b \frac{1}{4}\sin u\,du$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

6.4

7. $y = \frac{x^4}{8} + \frac{1}{4x^2}$ ($\frac{1}{4}x^{-2}$), $[1, 2]$

$y' = \frac{x^3}{2} - \frac{1}{2x^3}$

$\int_1^2 \sqrt{1 + \left(\frac{x^3}{2} - \frac{1}{2x^3}\right)^2}\, dx$

$= \int_1^2 \sqrt{1 + \frac{x^6}{4} - \frac{1}{2} + \frac{1}{4x^6}}\, dx$

$= \int_1^2 \sqrt{\frac{4x^6 + x^{12} - 2x^6 + 1}{4x^6}}\, dx$

$= \int_1^2 \frac{1}{2x^3}\sqrt{x^{12} + 2x^6 + 1}\, dx$

$= \int_1^2 \frac{1}{2x^3}\sqrt{(x^6+1)^2}\, dx$

$= \int_1^2 \frac{x^6+1}{2x^3}\, dx$

$= \int_1^2 \left(\frac{x^6}{2x^3} + \frac{1}{2x^3}\right) dx = \int_1^2 \left(\frac{1}{2}x^3 + \frac{1}{2}x^{-3}\right) dx$

$= \frac{1}{8}x^4 - \frac{1}{4x^2} \Big|_1^2$

$= 2 - \frac{1}{16} - \left(\frac{1}{8} - \frac{1}{4}\right)$

21.

$\int 2^{\sin x} \cos x\, dx = \int 2^u\, du$

$u = \sin x$

$du = \cos x\, dx$

$= \frac{1}{\ln 2} 2^{\sin x} + C$

20 $\int \frac{2^x}{2+2^x}\,dx = \int \frac{1}{\ln 2} \cdot \frac{du}{u}$

$u = 2 + 2^x$

$du = 2^x \ln 2\, dx$

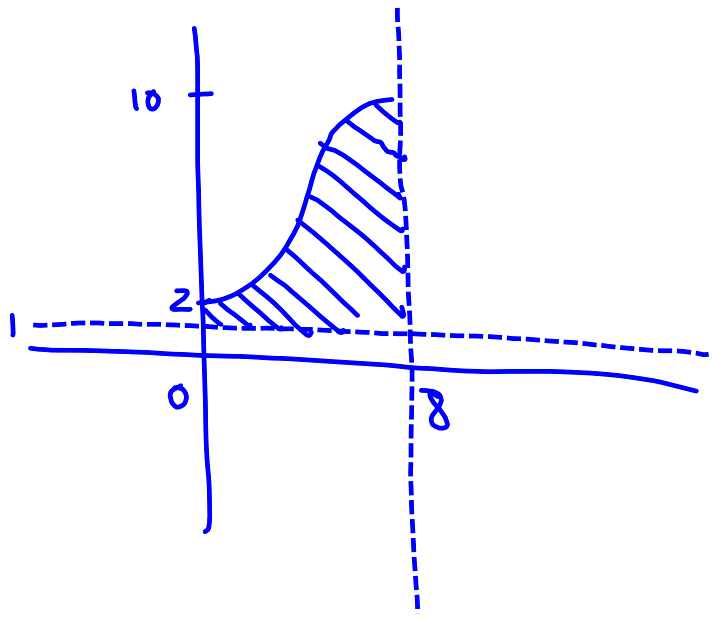
$\frac{1}{\ln 2} = 2^x\, dx$

$= \frac{1}{\ln 2} \cdot \ln\left|2+2^x\right| + C$

18 $\int x^2 e^{x^3/3}\,dx = \int e^u\,du = e^{x^3/3} + C$

$u = x^3/3$

$du = x^2\,dx$


10
2
1
0
8