- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts

7.2 Integration by Parts

$$\frac{d}{dx}[uv] = u \cdot \frac{d}{dx}[v] + v \cdot \frac{d}{dx}[u]$$

$$= uv' + vu'$$

Integrating both sides w.r.t. x yields:

$$uv = \int uv'dx + \int vu'dx$$

$$uv = \int u\,dv + \int v\,du$$

Rearranging yields:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int xe^x dx$$

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad \int dv = \int e^x dx$

$v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$= \boxed{xe^x - e^x + C}$$

6. $\int x^2 e^{2x} dx$

$u = x^2$ $\quad dv = e^{2x} dx$

$du = 2x\,dx$ $\quad \int dv = \int e^{2x} dx = \int \frac{1}{2} e^p dp$

$p = 2x$ $\quad = \frac{1}{2} e^p$

$dp = 2dx$ $\quad V = \frac{1}{2} e^{2x}$

$\frac{1}{2} dp = dx$

$$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int \frac{1}{2} e^{2x} (2x\,dx)$$

$$= \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$$

$u = x$ $\quad dv = e^{2x} dx$

$du = dx$ $\quad v = \frac{1}{2} e^{2x}$

$$= \frac{1}{2} x^2 e^{2x} - \left(\frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} dx\right)$$

$$= \boxed{\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C}$$

16. $\int x^4 \ln x \, dx = \frac{1}{5}x^5 \ln x - \int \left(\frac{1}{5}x^5\right) \cdot \frac{1}{x} \, dx$

$u = \ln x$ $\quad dv = x^4 \, dx$

$du = \frac{1}{x} \, dx$ $\quad \int dv = \int x^4 \, dx$

$v = \frac{1}{5}x^5$

$= \frac{1}{5}x^5 \ln x - \int \frac{1}{5}x^4 \, dx$

$= \boxed{\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C}$

$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx$

$u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} \, dx$ $\quad v = x$

$= x \ln x - \int dx$

$= \boxed{x \ln x - x + C}$

$\int \ln x = x \ln x - x + C$

34. $\int 4\arccos x\,dx$

$u = \arccos x \qquad dv = 4\,dx$

$du = \frac{-1}{\sqrt{1-x^2}}dx \qquad v = 4x$

$\int 4\arccos x\,dx = 4x\arccos x - \int \frac{-4x\,dx}{\sqrt{1-x^2}}$

$u = 1-x^2$

$du = -2x\,dx$

$2du = -4x\,dx$

$\stackrel{\vee}{=} 4x\arccos x - \int \frac{2du}{\sqrt{u}}$

$= 4x\arccos x - \int 2u^{-1/2}\,du$

$= 4x\arccos x - 4u^{1/2} + C$

$= \boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$

30. $\int x^2\cos x\,dx$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$\int x^2\cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$

$u = 2x \qquad dv = \sin x\,dx$

$du = 2dx \qquad v = -\cos x$

$\int x^2\cos x\,dx = x^2\sin x - \left(-2x\cos x - \int -2\cos x\,dx\right)$

$= x^2\sin x + 2x\cos x - \int 2\cos x\,dx$

$= \boxed{x^2\sin x + 2x\cos x - 2\sin x + c}$

36. $\int e^x \cos 2x \, dx$

$u = \cos 2x$ $\quad dv = e^x dx$

$du = -2\sin 2x dx$ $\quad v = e^x$

$\int e^x \cos 2x dx = e^x \cos 2x + \int 2e^x \sin 2x dx$

$u = 2\sin 2x$ $\quad dv = e^x dx$

$du = 4\cos 2x dx$ $\quad v = e^x$

$\int e^x \cos 2x dx = e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x dx$

$5\int e^x \cos 2x dx = e^x \cos 2x + 2e^x \sin 2x$

$\int e^x \cos 2x dx = \boxed{\frac{1}{5} e^x \cos 2x + \frac{2}{5} e^x \sin 2x + C}$