

- 7.1 #5-53 odd
- 7.2 #1-35 odd

basic integration techniques  
integration by parts

7.2

8.  $\int \ln 3x dx$

$$u = \ln 3x \quad dv = dx$$

$$du = \frac{1}{3x} \cdot 3 dx \quad v = x$$

$$\int \ln 3x dx = x \ln 3x - \int x \left( \frac{1}{3x} \cdot 3 dx \right)$$

$$= x \ln 3x - \int dx$$

$$= \boxed{x \ln 3x - x + C}$$

$$\int u dv = uv - \int v du$$

$$14. \int \frac{e^{1/t}}{t^2} dt = \int -e^u du$$

$$= \boxed{-e^{1/t} + c}$$

$$u = 1/t = t^{-1}$$

$$du = -t^{-2} dt$$

$$-du = \frac{1}{t^2} dt$$

$$48. \int_0^1 x^2 e^x dx = x^2 e^x - \int 2x e^x dx \Big|_0^1$$

$$u = x^2 \quad dv = e^x dx \quad \left| \begin{array}{l} p = 2x \quad dq = e^x dx \\ du = 2x dx \quad v = e^x \quad \left| \begin{array}{l} dp = 2 dx \quad q = e^x \end{array} \right. \end{array} \right. \Big|_0^1$$

$$\int_0^1 x^2 e^x dx = x^2 e^x - \left[ 2x e^x - \int 2e^x dx \right] \Big|_0^1$$

$$= x^2 e^x - 2x e^x + 2e^x \Big|_0^1$$

$$= (e - 2e + 2e) - (0 - 0 + 2)$$

$$= \boxed{e - 2}$$

$$28. \int x \sin x \, dx = -x \cos x - \int -\cos x \, dx$$

$$\left. \begin{array}{l} u = x \quad dv = \sin x \, dx \\ du = dx \quad v = -\cos x \end{array} \right\} = -x \cos x + \sin x + C$$

$$67. \int \frac{\ln x}{x} \, dx$$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} \, dx \end{aligned}$$

$$\int \frac{\ln x}{x} \, dx = \int u \, du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} (\ln x)^2 + C$$

$$68. \int x \ln x \, dx$$

$$\begin{aligned} u &= \ln x & dv &= x \, dx \\ du &= \frac{1}{x} \, dx & v &= \frac{1}{2} x^2 \end{aligned}$$

$$\begin{aligned} \int x \ln x \, dx &= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} \, dx \\ &= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx \end{aligned}$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$58. \int_0^{\pi/4} x \sec^2 x \, dx = x \tan x - \int \tan x \, dx \Big|_0^{\pi/4}$$

$$u = x \quad dv = \sec^2 x \, dx$$

$$du = dx \quad v = \tan x$$

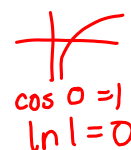
$$\int_0^{\pi/4} x \sec^2 x \, dx = x \tan x - \int \frac{\sin x}{\cos x} \, dx \Big|_0^{\pi/4}$$

$$p = \cos x$$

$$dp = -\sin x \, dx$$

$$\int_0^{\pi/4} x \sec^2 x \, dx = x \tan x + \int \frac{dp}{p} \Big|_{x=0}^{\pi/4}$$

$$= x \tan x + \ln |\cos x| \Big|_0^{\pi/4}$$



$$\cos 0 = 1$$

$$\ln 1 = 0$$

$$= \left( \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2} \right) - (0 + 0)$$

$$= \boxed{\frac{\pi}{4} + \ln \frac{\sqrt{2}}{2}}$$

40. Solve the differential equation.

$$\frac{dy}{dx} = x^2 \sqrt{x-1}$$

$$\int dy = \int x^2 \sqrt{x-1} \cdot dx$$

$$u = x-1 \rightarrow x = u+1$$

$$du = dx$$

$$= \int (u+1)^2 u^{1/2} \, du$$

$$= \int (u^2 + 2u + 1) u^{1/2} \, du$$

$$= \int (u^{5/2} + 2u^{3/2} + u^{1/2}) \, du$$

$$y = \frac{2}{7} (x-1)^{7/2} + \frac{4}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C$$

$$\frac{dy}{dx} = y'$$

$$y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

$$= \frac{df}{dx}$$

7.3 Trigonometric Integrals

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cot^2 x + 1 = \csc^2 x$$

$$= 1 - 2 \sin^2 x$$

$$4. \int \cos^3 x \sin^4 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \cdot \sin^4 x \, dx$$

$$= \int (1 - \sin^2 x) \cdot \sin^4 x \cdot \cos x \, dx$$

$$= \int (\sin^4 x - \sin^6 x) \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$= \int (u^4 - u^6) \, du$$

$$= \boxed{\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C}$$