- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts

$$\int u\,dv = uv - \int v\,du$$

7.2

8. $\int \ln 3x\,dx$

$u = \ln 3x \quad dv = dx$

$du = \frac{1}{3x} \cdot 3\,dx \quad v = x$

$$\int \ln 3x\,dx = x\ln 3x - \int x\left(\frac{1}{3x}\cdot 3\,dx\right)$$

$$= x\ln 3x - \int dx$$

$$= \boxed{x\ln 3x - x + C}$$

$$14.\ \int \frac{e^{1/t}}{t^2}\,dt = \int -e^u\,du$$

$$= -e^{1/t} + C$$

$u = 1/t = t^{-1}$

$du = -t^{-2}\,dt$

$-du = \frac{1}{t^2}\,dt$

$$48.\ \int_0^1 x^2 e^x\,dx = x^2e^x - \int 2xe^x\,dx \Big|_0^1$$

$u = x^2$ $\quad dv = e^x dx$ $\quad\Big|\quad p = 2x$ $\quad dq = e^x dx$

$du = 2x\,dx$ $\quad v = e^x$ $\quad\Big|\quad dp = 2dx$ $\quad q = e^x$

$$\int_0^1 x^2e^x\,dx = x^2e^x - \left[2xe^x - \int 2e^x\,dx\right]\Big|_0^1$$

$$= x^2e^x - 2xe^x + 2e^x \Big|_0^1$$

$$= (e - 2e + 2e) - (0 - 0 + 2)$$

$$= e - 2$$

28. $\int x\sin x\,dx = -x\cos x - \int -\cos x\,dx$

$u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$= -x\cos x + \sin x + C$

67. $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{\ln x}{x}\,dx = \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

68. $\int x\ln x\,dx$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{1}{2}x^2$

$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}\,dx$

$= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx$

$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

58. $\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x - \int \tan x\,dx \Big|_0^{\pi/4}$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x - \int \frac{\sin x}{\cos x}\,dx \Big|_0^{\pi/4}$

$p = \cos x$

$dp = -\sin x\,dx$

$\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x + \int \frac{dp}{p} \Big|_{x=0}^{\pi/4}$

$= x\tan x + \ln|\cos x| \Big|_0^{\pi/4}$

$\cos 0 = 1$

$\ln 1 = 0$

$= \left(\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}\right) - (0 + 0)$

$= \boxed{\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}}$

40. Solve the differential equation.

$$\frac{dy}{dx} = x^2\sqrt{x-1}$$

$\frac{dy}{dx} = y'$

$y = f(x)$

$\frac{dy}{dx} = f'(x) = \frac{df}{dx}$

$$\int dy = \int x^2\sqrt{x-1}\cdot dx$$

$u = x - 1 \rightarrow x = u + 1$

$du = dx$

$= \int (u+1)^2 u^{1/2}\,du$

$= \int (u^2 + 2u + 1)u^{1/2}\,du$

$= \int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right)du$

$$\boxed{y = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$$

7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

4. $\int \cos^3 x \sin^4 x \, dx$

$= \int \cos^2 x \cdot \cos x \cdot \sin^4 x \, dx$

$= \int (1 - \sin^2 x) \cdot \sin^4 x \cdot \cos x \, dx$

$= \int (\sin^4 x - \sin^6 x) \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int (u^4 - u^6) \, du$

$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$