- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts
- 7.3 #3-15odd; 21-37odd; 47-67odd — trigonometric integrals
- 7.4 #5-15odd; 19-43odd — trigonometric substitution

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\int u\,dv = uv - \int v\,du$

TEST # 4: Wed, 3 Feb

7.1 #39

$\log_a b^c = c\log_a b$

$$\int \frac{\ln x^2}{x}\,dx = \int \frac{2\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{dx}{x}$

...

43.

$$\int \frac{1}{\cos\theta - 1}\, d\theta$$

$$\int \frac{1}{\cos\theta - 1} \cdot \frac{\cos\theta + 1}{\cos\theta + 1}\, d\theta$$

$$= \int \frac{\cos\theta + 1}{\cos^2\theta - 1}\, d\theta$$

$$= \int \frac{\cos\theta + 1}{-\sin^2\theta}\, d\theta$$

$$= \int \frac{-\cos\theta}{\sin^2\theta}\, d\theta + \int \frac{-1}{\sin^2\theta}\, d\theta$$

$$\int -\csc\theta\cot\theta\, d\theta + \int -\csc^2\theta\, d\theta$$

$$\csc\theta + \cot\theta + C$$

25

$$\int \frac{x^2}{x-1}\, dx = \int \frac{(u+1)^2}{u}\, du = \int \frac{u^2+2u+1}{u}\, du$$

$u = x-1 \rightarrow x = u+1$

$du = dx$

$$= \int \left(u + 2 + \frac{1}{u}\right) du$$

$$= \frac{1}{2}u^2 + 2u + \ln|u| + C$$

$$= \boxed{\frac{1}{2}(x-1)^2 + 2(x-1) + \ln|x-1| + C}$$

7.2 #35

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$$

$u = e^{2x}$ $dv = \sin x\,dx$

$du = 2e^{2x}dx$ $v = -\cos x$

$u = 2e^{2x}$ $dv = \cos x\,dx$

$du = 4e^{2x}dx$ $v = \sin x$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$$

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x$$

$$\int e^{2x}\sin x\,dx = \boxed{-\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x + C}$$

12. $\int \sin^2 2x\,dx$

$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

Let $\theta = \frac{x}{2}$

$$\boxed{\cos x = 1 - 2\sin^2\frac{x}{2}}$$

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

using $\cos 2\theta = 2\cos^2\theta - 1$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 4x = 1 - 2\sin^2 2x$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$$\int \sin^2 2x\,dx = \int\left(\frac{1}{2} - \frac{1}{2}(\cos 4x)\right)dx$$

$u = 4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$= \frac{1}{2}x - \frac{1}{8}\int \cos u\,du$

$$\boxed{= \frac{1}{2}x - \frac{1}{8}\sin 4x + C}$$

26. $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) \, dx$

$= \boxed{\tan x - x + C}$

38. $\int \frac{\tan^2 x}{\sec^5 x} dx$

$= \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^5 x}{1} dx$

$= \int \sin^2 x \cdot \cos^3 x \, dx$

$= \int \sin^2 x \cos^2 x \underbrace{\cos x \, dx}_{du}$

$= \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^2 x - \sin^4 x) \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int (u^2 - u^4) \, du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

16. $\int x^2 \sin^2 x\,dx$

$\cos 2\theta = 1 - 2\sin^2\theta$
$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$u = x^2 \qquad dv = \sin^2 x\,dx$
$du = 2x\,dx \qquad v = \int \sin^2 x\,dx$
$= \int \frac{1-\cos 2x}{2}\,dx$
$= \frac{1}{2}x - \frac{1}{4}\sin 2x$

$\int x^2 \sin^2 x\,dx = x^2\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \int 2x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right)dx$
$= \frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \int x^2\,dx + \int \frac{1}{2}x\sin 2x\,dx$
$= \frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{3}x^3 + \int \frac{1}{2}x\sin 2x\,dx$

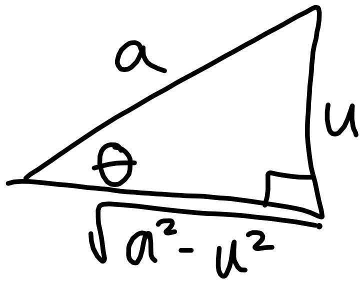
$u = \frac{1}{2}x \qquad dv = \sin 2x\,dx$
$du = \frac{1}{2}dx \qquad v = -\frac{1}{2}\cos 2x$

$= \frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \int \frac{1}{4}\cos 2x\,dx$
$= \boxed{\frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C}$

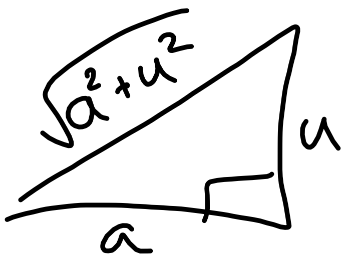
7.4 Trig Substitution

$\sqrt{a^2 - u^2} = a\cos\theta$
$\sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2\theta)}$
$u = a\sin\theta \qquad = \sqrt{a^2\cos^2\theta}$
$= a\cos\theta$

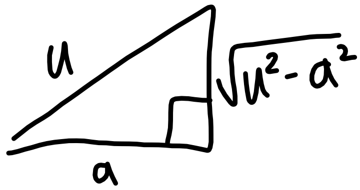
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sqrt{a^2 + u^2} = a\sec\theta$, $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$u = a\tan\theta$



$\sqrt{u^2 - a^2} = \begin{cases} a\tan\theta, & u > a \\ -a\tan\theta, & u < -a \end{cases}$
$u = a\sec\theta$
$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

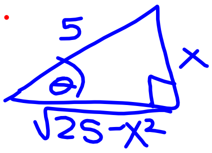


6. $$\int \frac{10}{x^2\sqrt{25-x^2}}\,dx = \int \frac{10\,dx}{(5\sin\theta)^2\sqrt{25-(5\sin\theta)^2}}$$

$x = 5\sin\theta$ $\quad \sin\theta = \frac{x}{5}$

$dx = 5\cos\theta\, d\theta$

$$= \int \frac{\overset{2}{\cancel{10}}\,dx}{\underset{5}{\cancel{25}}\sin^2\theta\sqrt{25(1-\sin^2\theta)}}$$



$$= \int \frac{2\,dx}{5\sin^2\theta\cdot\sqrt{25\cos^2\theta}}$$

$$= \int \frac{2\cdot \cancel{5}\cancel{\cos\theta}\,d\theta}{5\sin^2\theta\cdot \cancel{5}\cancel{\cos\theta}}$$

$$= \int \frac{2}{5}\csc^2\theta\, d\theta$$

$$= -\frac{2}{5}\cot\theta + C$$

$$= \boxed{\frac{-2\sqrt{25-x^2}}{5x} + C}$$