- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts
- 7.3 #3-15odd; 21-37odd; 47-67odd — trigonometric integrals
- 7.4 #5-15odd; 19-43odd — trigonometric substitution

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\boxed{\int u\,dv = uv - \int v\,du}$

TEST #4: Wed, 3 Feb

Sketch the region and set up the integral(s), but do not evaluate, to find the area of the region bounded by the graphs of the given functions.

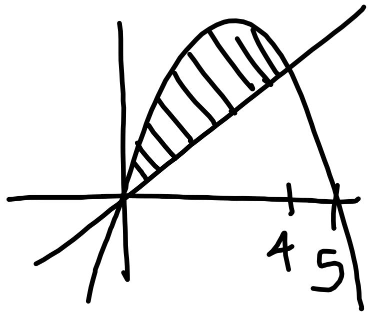
1. $y = 5x - x^2 = x(5-x)$, $y = x$

$5x - x^2 = x$

$0 = x^2 - 4x$

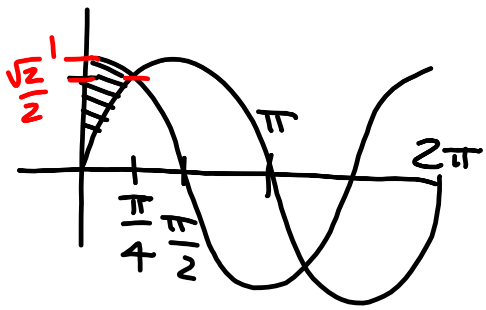
$0 = x(x-4)$

$x = 0, 4$



$$\int_0^4 (5x - x^2 - x)\,dx$$

2. $y = \cos x$, $y = \sin x$, $x = 0$, $x = \frac{\pi}{4}$



$$\int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$\int_0^{\sqrt{2}/2} \arcsin y\,dy + \int_{\sqrt{2}/2}^{1} \arccos y\,dy$$

Set up and simplify the integral, but do not evaluate, to find the arc length of the given function on the interval.

3. $y = \ln(\sin x)$, $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$y' = \frac{1}{\sin x} \cdot \cos x$$

$$y' = \cot x$$

$$= \int_{\pi/4}^{3\pi/4} \sqrt{1 + \cot^2 x}\, dx$$

$$= \int_{\pi/4}^{3\pi/4} \sqrt{\csc^2 x}\, dx$$

$$= \int_{\pi/4}^{3\pi/4} \csc x\, dx$$

Set up and simplify the integral, but do not evaluate, to find the surface area of the solid generated by revolving the graph of the equation about the x-axis.

4. $y = \frac{x^3}{6} + \frac{1}{2x}$, $[1,2]$

$$A = \int_a^b 2\pi r \sqrt{1 + [f'(x)]^2}\, dx$$

$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$A = \int_1^2 2\pi \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}\, dx$$

$$= \int_1^2 2\pi \left(\frac{x^4 + 3}{6x}\right) \sqrt{\cancel{1} + \frac{1}{4}x^4 + \frac{1}{2} + \frac{1}{4x^4}}\, dx$$

$$= \int_1^2 2\pi \left(\frac{x^4 + 3}{6x}\right) \sqrt{\frac{x^8 + 2x^4 + 1}{4x^4}}\, dx$$

$$= \int_1^2 2\pi \left(\frac{x^4 + 3}{6x}\right) \sqrt{\frac{(x^4 + 1)^2}{(2x^2)^2}}\, dx$$

$$= \int_1^2 \cancel{2}\pi \left(\frac{x^4 + 3}{6x}\right) \left(\frac{x^4 + 1}{\cancel{2}x^2}\right) dx$$

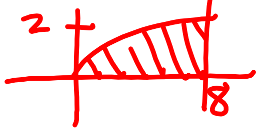
$$= \int_1^2 \pi\, \frac{x^8 + 4x^4 + 3}{6x^3}\, dx$$

$$= \int_1^2 \pi \left(\frac{x^5}{6} + \frac{2x}{3} + \frac{1}{2x^3}\right) dx$$

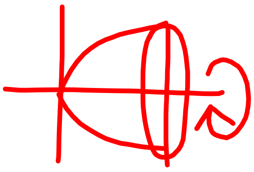
Sketch the region and set up the integrals for #5-10, but do not evaluate, to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines.

$y = \sqrt[3]{x}$, $y = 0$, $x = 0$, $x = 8$

$y^3 = x$

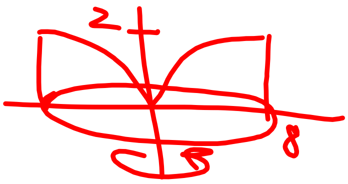


5. About the x-axis



$$\int_0^8 \pi\left(\sqrt[3]{x}\right)^2 dx$$

6. About the y-axis



$$\pi 8^2 \cdot 2 - \int_0^2 \pi\left(y^3\right)^2 dy$$

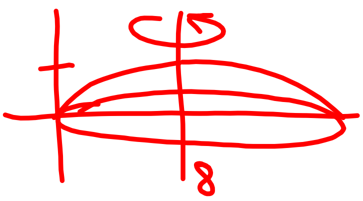
Sketch the region and set up the integrals for #5-10, but do not evaluate, to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines.

$y = \sqrt[3]{x}$, $y = 0$, $x = 0$, $x = 8$

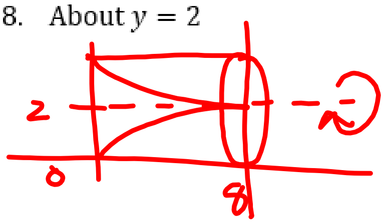
$y^3 = x$

7. About $x = 8$

$$\int_0^2 \pi\left(8 - y^3\right)^2 dy$$



8. About $y = 2$



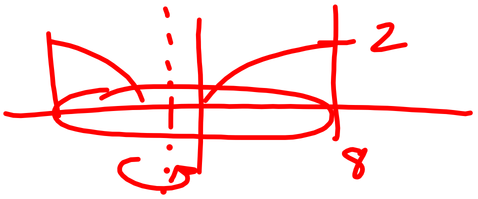
$$\pi 2^2 \cdot 8 - \int_0^8 \pi\left(2 - \sqrt[3]{x}\right)^2 dx$$

Sketch the region and set up the integrals for #5-10, but do not evaluate, to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines.

$y = \sqrt[3]{x}$, $y = 0$, $x = 0$, $x = 8$

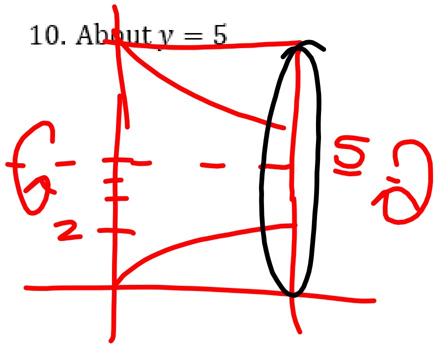
$y^3 = x$

9. About $x = -1$



$$\pi \cdot 9^2 \cdot 2 - \int_0^2 \pi (y^3 + 1)^2 \, dy$$

10. About $y = 5$



$$\pi \cdot 5^2 \cdot 8 - \int_0^8 \pi (5 - \sqrt[3]{x})^2 \, dx$$

11. Find $F'(x)$ if $f(x) = \int_{\pi/4}^{x^2} \cos x \, dx$.

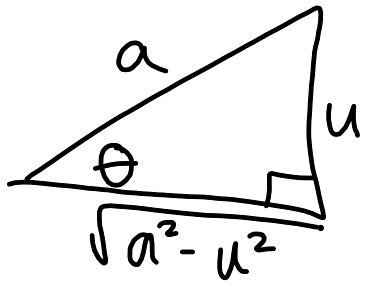
$$F'(x) = \cos(x^2) \cdot 2x$$

7.4 Trig Substitution

$\sqrt{a^2-u^2} = a\cos\theta$

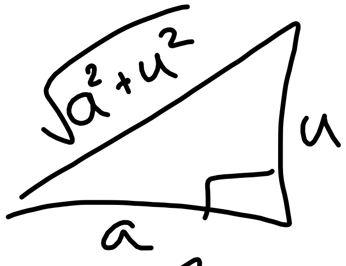
$\sqrt{a^2 - a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta$

$u = a\sin\theta$ $\qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sqrt{a^2+u^2} = a\sec\theta$ $\quad , -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

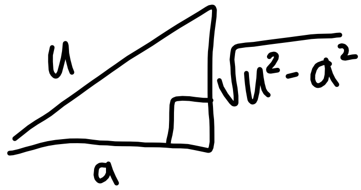
$u = a\tan\theta$



$\sqrt{u^2-a^2} = \begin{cases} +a\tan\theta, & u>a \\ -a\tan\theta, & u<-a \end{cases}$

$u = a\sec\theta$

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$



12. $\int \frac{x^3\,dx}{\sqrt{x^2-4}} = \int \frac{(2\sec\theta)^3 \cdot 2\sec\theta\tan\theta\,d\theta}{\sqrt{(2\sec\theta)^2 - 2^2}} =$

$\sqrt{4}$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\int \frac{16\sec^4\theta\tan\theta\,d\theta}{\sqrt{4(\sec^2\theta-1)}} = \int \frac{16\sec^4\theta\tan\theta\,d\theta}{2\sqrt{\tan^2\theta}}$

$= \int \frac{8\sec^4\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \int 8\sec^4\theta\,d\theta$

$= \int 8(1+\tan^2\theta)\sec^2\theta\,d\theta = \int 8(1+u^2)\,du$

Let $u = \tan\theta$

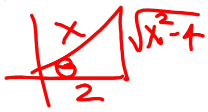
$du = \sec^2\theta\,d\theta$

$= 8u + \frac{8}{3}u^3 + C$

$= 8\tan\theta + \frac{8}{3}\tan^3\theta + C$

$x = 2\sec\theta$

$\frac{x}{2} = \sec\theta$



$= \boxed{8\cdot\frac{\sqrt{x^2-4}}{2} + \frac{8}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3 + C}$

16. $\displaystyle\int \frac{x^2\,dx}{(1+x^2)^2} = \int \frac{\tan^2\theta \cdot \sec^2\theta\, d\theta}{(1+\tan^2\theta)^2}$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$= \int \frac{\tan^2\theta\, \sec^2\theta\, d\theta}{(\sec^2\theta)^2}$$

$$= \int \frac{\tan^2\theta}{\sec^2\theta}\, d\theta$$

$$= \int \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1}\, d\theta = \int \sin^2\theta\, d\theta$$

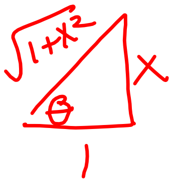
$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$\frac{x}{1} = \tan\theta$



$$= \frac{1}{2}\arctan x - \frac{1}{4}\,\frac{x}{\sqrt{1+x^2}}\,\frac{1}{\sqrt{1+x^2}} + C$$

$$= \boxed{\frac{1}{2}\arctan x - \frac{x}{4(1+x^2)} + C}$$

$\sin 2\theta =$
$2\sin\theta\cos\theta$