- 7.1 #5-53 odd — basic integration techniques
- 7.2 #1-35 odd — integration by parts
- 7.3 #3-15odd; 21-37odd; 47-67odd — trigonometric integrals
- 7.4 #5-15odd; 19-43odd — trigonometric substitution
- 7.5 #15-27 odd — partial fractions
- 5.7 #55, 57, 59 — separation of variables
- 6.5 #9,11,39,40 — work

Test #4 - Wed. 3 Feb

16. $\int \frac{x^2 dx}{(1+x^2)^2}$

$dv = \frac{x}{(1+x^2)^2} dx$ $\quad u = x$

$du = dx$

$p = 1 + x^2$

$dp = 2x\,dx$

$\frac{1}{2} dp = x\,dx$

$v = \int \frac{\frac{1}{2} dp}{p^2}$

$v = -\frac{1}{2p}$

$= \frac{-1}{2(1+x^2)}$

$-\frac{x}{2(1+x^2)} - \int \frac{-1}{2(1+x^2)} dx$

$\frac{-x}{2(1+x^2)} + \frac{1}{2}\arctan x + C$

30. $\int \frac{dx}{x\sqrt{4x^2+16}} = \int \frac{dx}{x\sqrt{4(x^2+4)}} = \int \frac{dx}{2x\sqrt{x^2+2^2}}$

$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$

Let $x = 2\tan\theta$
$dx = 2\sec^2\theta\, d\theta$

$\to = \int \frac{2\sec^2\theta\, d\theta}{2(2\tan\theta)\sqrt{(2\tan\theta)^2+4}}$

$= \int \frac{\sec^2\theta\, d\theta}{2\tan\theta\sqrt{4\tan^2\theta+4}} = \int \frac{\sec^2\theta\, d\theta}{2\tan\theta\sqrt{4(\tan^2\theta+1)}}$

$= \int \frac{\sec^2\theta\, d\theta}{2\tan\theta \cdot 2 \cdot \sqrt{\sec^2\theta}} = \int \frac{\sec^2\theta\, d\theta}{4\tan\theta\sec\theta}$

$= \int \frac{\sec\theta\, d\theta}{4\tan\theta} = \int \frac{1}{4} \cdot \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta}\, d\theta$

$= \int \frac{1}{4}\csc\theta\, d\theta$ $\cancel{= \frac{1}{4}\theta\csc\theta - \int -\frac{1}{4}\theta\csc\theta\cot\theta\, d\theta}$

$\cancel{u = \frac{1}{4}\csc\theta,\ dv = d\theta}$
$\cancel{du = -\frac{1}{4}\csc\theta\cot\theta\, d\theta,\ v = \theta}$

$\cancel{\int \frac{1}{4\sin\theta} \cdot \frac{\sin\theta}{\sin\theta}\, d\theta = \int \frac{\sin\theta\, d\theta}{4\sin^2\theta} = \int \frac{\sin\theta\, d\theta}{4(1-\cos^2\theta)}}$

$\cancel{u = \cos\theta}$
$\cancel{du = -\sin\theta\, d\theta}$
$\cancel{= \int \frac{-du}{4(1-u^2)}}$

$\int \frac{\csc\theta\, d\theta}{4} \cdot \frac{\csc\theta + \cot\theta}{\csc\theta + \cot\theta}$

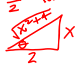
$= \int \frac{\csc^2\theta + \csc\theta\cot\theta\, d\theta}{4(\csc\theta + \cot\theta)} = \int \frac{-du}{4u}$

$u = \csc\theta + \cot\theta$
$du = (-\csc\theta\cot\theta - \csc^2\theta)\, d\theta$
$= -(\csc\theta\cot\theta + \csc^2\theta)\, d\theta$

$\int = -\frac{1}{4}\ln|\csc\theta + \cot\theta| + C$

$= \boxed{-\frac{1}{4}\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{2}{x}\right| + C}$

$x = 2\tan\theta$
$\frac{x}{2} = \tan\theta$



40. $\int x \arcsin x\, dx =$

$u = \arcsin x$ $\qquad dv = x\, dx$
$du = \frac{1}{\sqrt{1-x^2}}\, dx$ $\qquad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \arcsin x - \int \frac{\frac{1}{2}x^2}{\sqrt{1-x^2}}\, dx$

Let $x = \sin\theta$
$dx = \cos\theta\, d\theta$

$= \frac{1}{2}x^2 \arcsin x - \int \frac{\frac{1}{2}\sin^2\theta \cdot \cancel{\cos\theta}\, d\theta}{\boxed{\sqrt{1-\sin^2\theta}}} \to \sqrt{\cos^2\theta} = \cancel{\cos\theta}$

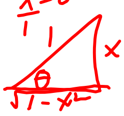
$= \frac{1}{2}x^2 \arcsin x - \int \frac{1}{2}\sin^2\theta\, d\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$
$2\sin^2\theta = 1 - \cos 2\theta$
$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \frac{1}{2}x^2 \arcsin x - \int \frac{1}{4}(1 - \cos 2\theta)\, d\theta$

$\arcsin x = \theta$
$x = \sin\theta$

$= \frac{1}{2}x^2 \arcsin x - \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + C$



$= \frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{\cancel{8}_4} \cdot \cancel{2}\sin\theta\cos\theta + C$

$= \boxed{\frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C}$

6. $\int e^x \cos x \, dx$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \sin x$

$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x dx \qquad v = -\cos x$

$\int e^x \cos x \, dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x \, dx \right]$

$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$

$$\int e^x \cos x \, dx = \boxed{\frac{e^x \sin x + e^x \cos x}{2} + C}$$

$$F(x) = \int_a^{g(x)} f(x) \, dx$$

$$F'(x) = f(g(x)) \cdot g'(x)$$