- 7.1 #5-53 odd   basic integration techniques
- 7.2 #1-35 odd   integration by parts
- 7.3 #3-15odd; 21-37odd; 47-67odd   trigonometric integrals
- 7.4 #5-15odd; 19-43odd   trigonometric substitution
- 7.5 #15-27 odd   partial fractions
- 5.7 #55, 57, 59   separation of variables
- 6.5 #9,11,39,40   work

Test #4 - Wed. 3 Feb

7.5 Partial Fractions

$$\int \frac{1}{x^2-5x+6} \, dx$$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$(x-3)(x-2)$$

$$= \frac{A(x-2) + B(x-3)}{(x-3)(x-2)}$$

$$= \frac{(A+B)x + (-2A-3B)}{x^2-5x+6}$$

$$A+B=0 \rightarrow A=-B$$

$$-2A-3B=1$$

$$-2(-B)-3B=1$$

$$-B=1$$

$$B=-1$$

$$A=1$$

$$\int \frac{1}{x^2-5x+6} \, dx = \int \left(\frac{1}{x-3} + \frac{-1}{x-2}\right) dx$$

$$= \ln|x-3| - \ln|x-2| + C$$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}dx$$

$$x(x^2+2x+1)$$
$$x(x+1)(x+1)$$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

$$= \frac{Ax^2+2Ax+A+Bx^2+Bx+Cx}{x(x+1)^2}$$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{(A+B)x^2+(2A+B+C)x+A}{x(x+1)^2}$$

$$5=A+B \rightarrow B=5-A=5-6=-1$$
$$20=2A+B+C \rightarrow C=20-2A-B$$
$$6=A \qquad\qquad =20-2(6)-(-1)$$
$$\qquad\qquad\qquad =9$$

$$\int \frac{5x^2+20x+6}{x(x+1)^2}dx = \int\left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\right)dx$$

$$\boxed{= 6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C}$$

$$\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}dx$$

$$\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}$$

$$= \frac{A(x-1)(x^2+4)+Bx(x^2+4)+(Cx+D)x(x-1)}{x(x-1)(x^2+4)}$$

$$= \frac{A(x^3-x^2+4x-4)+Bx^3+4Bx+Cx^3+Dx^2-Cx^2-Dx}{x(x-1)(x^2+4)}$$

$$\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{Ax^3-Ax^2+4Ax-4A+Bx^3+4Bx+Cx^3+Dx^2-Cx^2-Dx}{x(x-1)(x^2+4)}$$

$$= \frac{x^3(A+B+C)+x^2(-A+D-C)+x(4A+4B-D)-4A}{x(x-1)(x^2+4)}$$

$$2=A+B+C$$
$$0=-A+D-C \rightarrow C=A+D=2+4$$
$$-4=4A+4B-D \qquad C=2$$
$$-8=-4A \rightarrow A=2$$

$$2=B+D \rightarrow D=2-B=2-(-2)=4$$
$$D=4 \qquad 2=B+D$$
$$-4-4(2)=4B-D \rightarrow -12=4B-D$$
$$-10=5B$$
$$-2=B$$

$$\int \frac{2x^3-4x-8}{x(x-1)(x^2+4)}dx = \int\left(\frac{2}{x} + \frac{-2}{x-1} + \frac{2x+4}{x^2+4}\right)dx$$

$$= 2\ln|x| - 2\ln|x-1| + \int\left(\frac{2x}{x^2+4} + \frac{4}{x^2+4}\right)dx$$

$$u=x^2+4$$
$$du=2x\,dx \qquad a=2$$

$$\boxed{= 2\ln|x| - 2\ln|x-1| + \ln|x^2+4| + 2\arctan\frac{x}{2} + C}$$