- **5.7 #55, 57, 59** — **separation of variables**
- **6.5 #9,11,39,40** — **work**

Final Exam: Tues, 16 Feb, 9-11am
Can replace your 2nd lowest test grade!

1. Evaluate the indefinite integral.

$\int \frac{1}{x^2-6x+8}\,dx =$

(A) $\boxed{\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C}$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right|+C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right|+C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right|+C$

(E) $\ln\left|(x-2)(x-4)\right|+C$

$$\int \frac{1}{(x-4)(x-2)}\,dx = \int \frac{1}{2}\left(\frac{1}{x-4}\right)dx + \int -\frac{1}{2}\left(\frac{1}{x-2}\right)dx$$

$$= \frac{1}{2}\ln|x-4| - \frac{1}{2}\ln|x-2| + C$$

$$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$$

$$= \frac{A(x-2)+B(x-4)}{(x-4)(x-2)}$$

$$= \frac{(A+B)x+(-2A-4B)}{(x-4)(x-2)}$$

$A+B=0$

$-2A-4B=1$

$A=-B$

$-2(-B)-4B=1$

$-2B=1$

$B=-\frac{1}{2}$ $A=\frac{1}{2}$

2. Evaluate the indefinite integral.

$\int x^2 \sin x\, dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$

(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$

(C) $\boxed{-x^2 \cos x + 2x \sin x + 2\cos x + C}$

(D) $-\frac{x^3}{3} \cos x + C$

(E) $2x \cos x + C$

$u = x^2 \quad dv = \sin x\, dx$

$du = 2x\, dx \quad v = -\cos x$

$= -x^2 \cos x - \int -2x \cos x\, dx$

$= -x^2 \cos x + \int 2x \cos x\, dx$

$u = 2x \quad dv = \cos x\, dx$

$du = 2\, dx \quad v = \sin x$

$= -x^2 \cos x + 2x \sin x - \int 2 \sin x\, dx$

$= -x^2 \cos x + 2x \sin x - (-2\cos x) + C$

3. Use the 2nd Fundamental Theorem of Calculus to evaluate the derivative of the function at the given value.

If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) $\boxed{3}$ (E) 18

$F'(x) = \sqrt{x^3 + 1} \cdot \underbrace{(x')}_{= 1}$

$F'(2) = \sqrt{2^3 + 1}$

$= \sqrt{9} = 3$

4. Evaluate the indefinite integral.

$\int xe^{2x}dx =$ $\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(A) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$ (B) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{2} + C$ (C) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{2} + C$ (E) $\frac{x^2e^{2x}}{4} + C$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \int e^{2x}dx$

$= \frac{1}{2}e^{2x}$

5. Evaluate the indefinite integral.

$\int \sin^2 x \, dx$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \int \frac{1}{2}dx - \int \frac{\cos 2x}{2}dx$

$= \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

6. Evaluate the indefinite integral.

$$\int e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x dx \qquad v = \sin x$

$$\int \quad = e^x \sin x - \int e^x \sin x \, dx$$

$u = e^x \qquad dv = \sin x\,dx$

$du = e^x dx \qquad v = -\cos x$

$$\int e^x \cos x\,dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x\,dx\right]$$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x\,dx = \boxed{\frac{e^x \sin x + e^x \cos x}{2} + C}$$

7. Evaluate the indefinite integral using the given substitution.

$$\int x^3\sqrt{x^2-4}\,dx \quad , \quad x = 2\sec\theta$$

$dx = 2\sec\theta\tan\theta\,d\theta$

$$\int (2\sec\theta)^3 \underbrace{\sqrt{(2\sec\theta)^2 - 4}} \cdot 2\sec\theta\tan\theta\,d\theta$$

$\sqrt{4\sec^2\theta - 4}$

$\sqrt{4(\sec^2\theta - 1)}$

$\sqrt{4\tan^2\theta}$

$2\tan\theta$

$$= \int 8\sec^3\theta \cdot 2\tan\theta \cdot 2\sec\theta\tan\theta\,d\theta$$

$$= \int 32\tan^2\theta\sec^2\theta\sec^2\theta\,d\theta$$

$$= \int 32\tan^2\theta(1+\tan^2\theta)\sec^2\theta\,d\theta$$

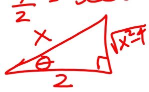
$$= \int 32(\tan^2\theta + \tan^4\theta)\sec^2\theta\,d\theta$$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta$

$$= \int 32(u^2 + u^4)\,du$$

$x = 2\sec\theta$

$\frac{x}{2} = \sec\theta$



$$= \frac{32}{3}u^3 + \frac{32}{5}u^5 + C$$

$$= \frac{32}{3}(\tan\theta)^3 + \frac{32}{5}(\tan\theta)^5 + C$$

$$= \frac{32}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3 + \frac{32}{5}\left(\frac{\sqrt{x^2-4}}{2}\right)^5 + C$$

$$= \boxed{\frac{4}{3}\sqrt{(x^2-4)^3} + \frac{1}{5}\sqrt{(x^2-4)^5} + C}$$

5.7 Solving Differential Equations by Separation of Variables

$y = 5x^3 - \cos x$

what is the differential of y?

$dy = (15x^2 + \sin x)dx$

$y' = \frac{dy}{dx} = \frac{d}{dx}(y) = 15x^2 + \sin x$

5.7 Ex 3 - Find the general solution.

$(x^2+4)\frac{dy}{dx} = xy$

$(x^2+4)dy = xy\,dx$

$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2+4}$

$\ln|y| = \int \frac{x\,dx}{x^2+4}$ $u = x^2+4$, $du = \cancel{2x}dx$, $\frac{1}{2}du = x\,dx$

$\ln|y| = \int \frac{\frac{1}{2}du}{u}$

$\ln|y| = \frac{1}{2}\ln|x^2+4| = \ln(x^2+4)^{1/2}$

$\ln|y| = \ln\sqrt{x^2+4}$

$|y| = \sqrt{x^2+4}$

$\boxed{y = \pm\sqrt{x^2+4} + C}$

Ex 4 Find a particular solution.

$$xy\,dx + e^{-x^2}(y^2-1)\,dy = 0 \qquad ; \quad y(0)=1$$

$$\frac{x\cancel{y}\,dx}{\cancel{y}(e^{-x^2})} = \frac{-\cancel{e^{-x^2}}(y^2-1)\,dy}{y(-\cancel{e^{-x^2}})}$$

$$\int -xe^{x^2}\,dx = \int \frac{y^2-1}{y}\,dy$$