- **5.7 #55, 57, 59** — **separation of variables**
- **6.5 #9,11,39,40** — **work**

Final Exam: Tues, 16 Feb, 9-11am
Can replace your 2nd lowest test grade!

Ex 4 Find a particular solution.

$$xy\,dx + e^{-x^2}(y^2-1)\,dy = 0 \quad ; \; y(0)=1$$

$$\frac{x\cancel{y}\,dx}{\cancel{y}(e^{-x^2})} = \frac{-\cancel{e^{-x^2}}(y^2-1)\,dy}{y(-\cancel{e^{-x^2}})}$$

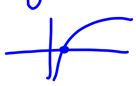
$$\int -xe^{x^2}\,dx = \int \frac{y^2-1}{y}\,dy$$

$u = x^2$
$du = 2x\,dx$
$-\frac{1}{2}du = -x\,dx$

$$\int -\frac{1}{2}e^{u}\,du = \int \left(y - \frac{1}{y}\right)dy$$

$$\boxed{-\frac{1}{2}e^{x^2} = \frac{1}{2}y^2 - \ln|y| + C}$$ general solution

$y(0)=1$



$$-\frac{1}{2}e^{0^2} = \frac{1}{2}(1)^2 - \ln|1| + C$$

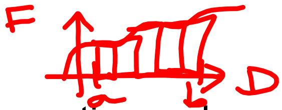
$$-\frac{1}{2} = \frac{1}{2} - 0 + C$$

$$-1 = C$$

$$\boxed{-\frac{1}{2}e^{x^2} = \frac{1}{2}y^2 - \ln|y| - 1}$$ particular solution

6.5 - Work

If an object is moved a distance D in the direction of an applied constant force F, then the work W done by the force is defined as W=FD.



If an object is moved along a straight line by a continuously varying force F(x), then the work W done by the force as the object is moved from x=a to x=b is

$$W = \int_a^b F(x)\,dx$$

6.5

work done by an expanding gas

initial volume: 1 ft^3

initial pressure: 500 pounds per ft^2

gas expands to a volume of $2 ft^3$

Find the work done by the gas.

Assume pressure is inversely proportional to volume.

$P = \frac{K}{V}$ since $500 = \frac{K}{1}$, $K = 500$

$$W = \int_{V_0}^{V_1} \frac{K}{V}\,dV = \int_1^2 \frac{500}{V}\,dV = 500\ln|V|\Big|_1^2$$

$$= 500\ln 2 \approx \boxed{346.6 \text{ foot·pounds}}$$

Compressing a spring

A force of 750 lb compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring additional 3 in.

Hooke's Law: $F(x) = kx \Rightarrow F(x) = 250x$

$750 = k \cdot 3$

$250 = k$

$$W = \int_3^6 250x\,dx = 125x^2 \Big|_3^6 = 125(36) - 125(9)$$

$$125(27) = \boxed{3375 \text{ inch-pounds}}$$

$$\sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right) \cdot \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right) \cdot \frac{b-a}{n}$$

Review

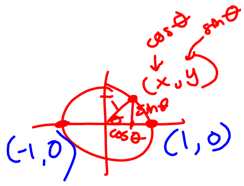
Determine

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$$

Area Between Curves

$f(x) = \sin x + 5$ $0 \le x \le 5\pi$

$g(x) = -\sin x + 5$

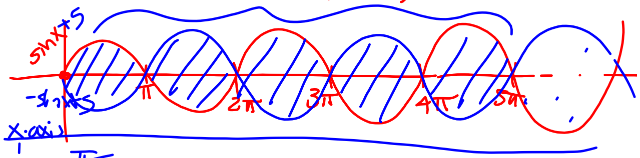


$\sin x + 5 = -\sin x + 5$

$2\sin x = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$



$$5\int_0^{\pi}\left[(\sin x+5)-(-\sin x+5)\right]dx$$

$$= 5\int_0^{\pi} 2\sin x\,dx = -10\cos x\Big|_0^{\pi}$$

$$= -10\cos\pi - (-10\cos 0)$$

$$= -10(-1) - (-10\cdot 1)$$

$$= 10 + 10$$

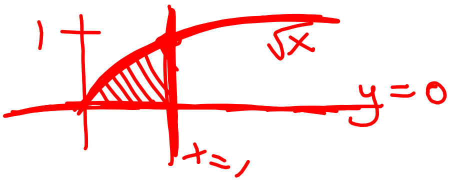
$$= \boxed{20}$$

Volume of solid of revolution

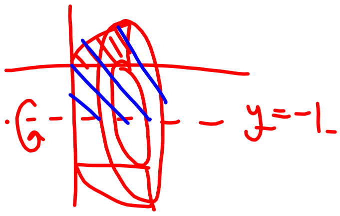
$f(x) = \sqrt{x} \rightarrow y = \sqrt{x}$

$y = 0 \qquad y^2 = x$

$x = 1$



rot @ $y=-1$ @ $x=-2$



$$V = \int_0^1 \pi\left(\sqrt{x} - (-1)\right)^2 \cdot dx - \pi(1)^2 \cdot 1$$

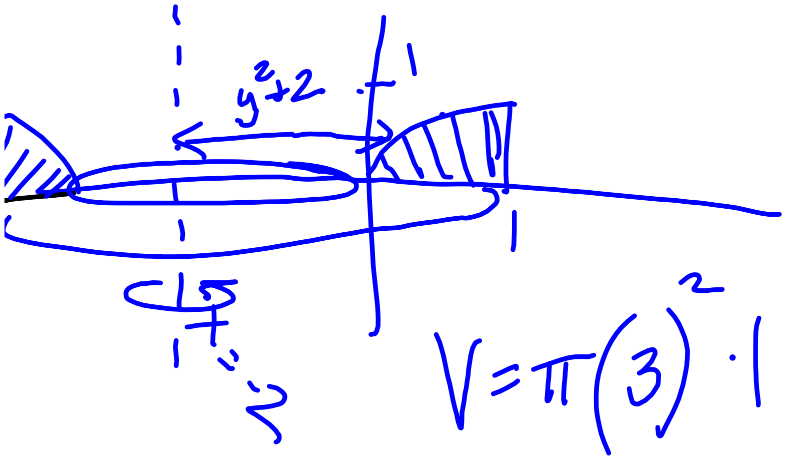
$$= \int_0^1 \pi(x + 2\sqrt{x} + 1)\,dx - \pi$$

$$= \int_0^1 (\pi x + 2\pi x^{1/2} + \pi)\,dx - \pi$$

$$= \frac{\pi x^2}{2} + \frac{4}{3}\pi x^{3/2} + \pi x \Big|_0^1 - \pi$$

$$= \frac{\pi}{2} + \frac{4\pi}{3} + \cancel{\pi} - \cancel{\pi}$$

$$= \frac{3\pi + 8\pi}{6} = \boxed{\frac{11\pi}{6}}$$



$$V = \pi(3)^2 \cdot 1 - \int_0^1 \pi(y^2+2)^2 \cdot dy$$