- **5.7 #55, 57, 59** — **separation of variables**
- **6.5 #9,11,39,40** — **work**

Final Exam: Tues, 16 Feb, 9-11am
Can replace your 2nd lowest test grade!

If you are NOT taking AP Calculus Review Spring term, bring your textbook to the final exam. There is a different textbook for Differential Equations.

At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

$v(0) = 0$
$a(t) = 24t^2$
$s(2) - s(0)$

$v(t) = \int 24t^2\,dt = 8t^3 + c_1$
$v(t) = 8t^3 + c_1 = 8t^3$
$0 = 8(0)^3 + c_1 \rightarrow c_1 = 0$
$s(t) = \int 8t^3\,dt = 2t^4 + c_2$

$s(2) - s(0) = 2(2)^4 + c_2 - (2(0)^4 + c_2)$
$= \boxed{32 \text{ ft}}$

The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

avg. value of f(x) on [a,b] $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{2-0}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\int_0^2 x^{1/2}\,dx = \frac{1}{2}\left(\frac{2}{3}x^{3/2}\right)\Big|_0^2$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(2)^{3/2} - \frac{1}{2}\cdot\frac{2}{3}(0)^{3/2}$$

$$= \frac{2\sqrt{2}}{3}$$

What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$ (B) 4 (C) $\frac{14}{3}$ (D) $\frac{16}{3}$ (E) 7

$$\int_0^3 \sqrt{1 + \left[\left(\frac{2}{3}x^{3/2}\right)'\right]^2}\,dx = \int_0^3 \sqrt{1+x}\,dx$$

$u = 1 + x$

$du = dx$

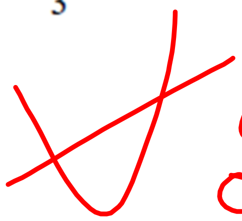
$$= \int_{x=0}^{3} \sqrt{u}\,du = \int_1^4 u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2}\Big|_1^4 = \frac{2}{3}\sqrt{4}^3 - \frac{2}{3}\sqrt{1}^3$$

$$= \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\boxed{\frac{4}{3}}$ (D) 2 (E) $\frac{14}{3}$



$x = x^2 - 3x + 3$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1, 3$

$\int_1^3 (x - (x^2 - 3x + 3))\,dx$

$= \int_1^3 (-x^2 + 4x - 3)\,dx$

$= -\frac{1}{3}x^3 + 2x^2 - 3x \Big|_1^3$

$= \cancel{-9 + 18 - 9} - \left(\frac{-1}{3} + 2 - 3\right)$

$= \frac{1}{3} + \frac{3}{3} = \frac{4}{3}$

$\int_1^4 |x - 3|\,dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\boxed{\frac{5}{2}}$ (D) $\frac{9}{2}$ (E) 5

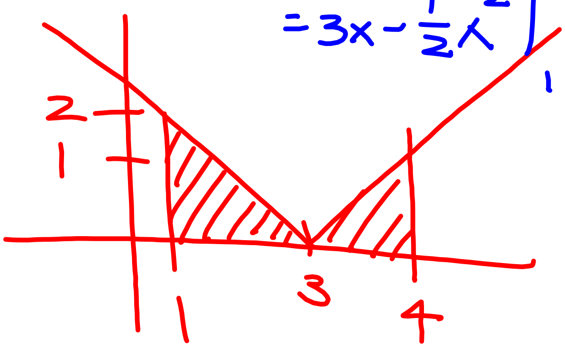
$|x-3| = \begin{cases} x-3, & x-3>0,\ x>3 \\ -(x-3), & x<3 \end{cases}$

$\int_1^4 |x-3|\,dx = \int_1^3 (3-x)\,dx + \int_3^4 (x-3)\,dx$

$= 3x - \frac{1}{2}x^2 \Big|_1^3 + \frac{1}{2}x^2 - 3x \Big|_3^4$

$= 2 + \frac{1}{2}$

$= 9 - \frac{9}{2} - 3 + \frac{1}{2} + 8 - 12 - \frac{9}{2} + 9$



If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

$\boxed{\text{(A)} \quad e^{\tan x} + 4}$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$

(D) $\tan x + 5$ (E) $\tan x + 5e^x$

$$\int \frac{dy}{y} = \int \sec^2 x \, dx$$

$$\ln|y| = \tan x + c_1$$

$$e^{\ln|y|} = e^{\tan x} + c_2$$

$$|y| = e^{\tan x} + c$$

$$5 = e^{\tan 0} + c$$

$$5 = 1 + c$$

$$4 = c$$

An antiderivative for $\frac{1}{x^2 - 2x + 2}$ is

(A) $-(x^2 - 2x + 2)^{-2}$

(B) $\ln(x^2 - 2x + 2)$

(C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

$\boxed{\text{(E)} \quad \arctan(x-1)}$

$$\frac{1}{x^2 - 2x + 1 + 1} = \frac{1}{(x-1)^2 + 1}$$

$$\int \frac{1}{x^2 - 2x + 2} \, dx$$

Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

$$\int \sin x \cos x \, dx$$

$$= \int \frac{1}{2} \cdot \underline{2\sin x \cos x} \, dx$$

$$= \int \frac{1}{2} \sin 2x \, dx$$

$u = 2x$
$du = 2dx$
$\frac{1}{2} du = dx$

$$= -\frac{1}{4}\cos 2x$$

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$-\frac{1}{4}(2\cos^2 x - 1) \qquad -\frac{1}{4}(1 - 2\sin^2 x)$$

$$= -\frac{1}{2}\cos^2 x + \frac{1}{4} \qquad -\frac{1}{4} + \frac{1}{2}\sin^2 x$$

A force of 10 pounds is required to stretch a spring 4 inches beyond its natural length. Assuming Hooke's law applies, how much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

$F = kx \qquad 10 = k \cdot 4 \qquad k = \frac{5}{2}$

(A) 60.0 inch-pounds
(B) 45.0 inch-pounds
(C) 40.0 inch-pounds
(D) 15.0 inch-pounds
(E) 7.2 inch-pounds

$$W(x) = \int_0^6 \frac{5}{2} x \, dx$$

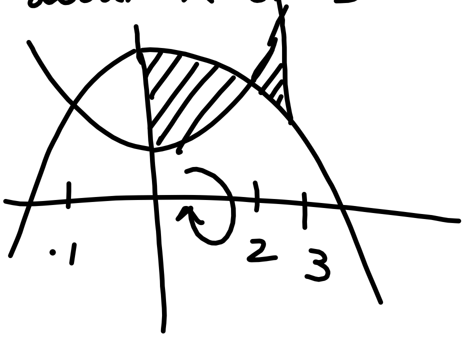
$$= \frac{5}{2} \cdot \frac{1}{2} x^2 \Big|_0^6$$

$$= \frac{5}{4} \cdot 36 - \cancel{\frac{5}{4} \cdot 0}$$

Volume

$y=x^2+1$, $y=-x^2+2x+5$, $x=0$, $x=3$

about x-axis



$$x^2+1=-x^2+2x+5$$
$$2x^2-2x-4=0$$
$$2(x^2-x-2)=0$$
$$2(x-2)(x+1)=0$$
$$x=-1, 2$$

$$\int_0^2 \pi(-x^2+2x+5)^2\,dx - \int_0^2 \pi(x^2+1)^2\,dx + \int_2^3 \pi(x^2+1)^2\,dx - \int_2^3 \pi(-x^2+2x+5)^2\,dx$$

Surface of Revolution

$2\pi r h$, $h=$ arclength

64

36. $y=\frac{x}{2}$, $[0,6]$ about x-axis

$$\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

5.7

59. $y(1+x^2)y' - x(1+y^2) = 0$; $y(0) = \sqrt{3}$

57. $y(x+1) + y' = 0$; $y(-2) = 1$

55. $yy' - e^x = 0$; $y(0) = 4$

15. $\displaystyle\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\,dx$

17. $\displaystyle\int \frac{4x^2+2x-1}{x^3+x^2}\,dx$

$x^2(x+1)$

$$\int \frac{4x^2+2x-1}{x^2(x+1)}\,dx = \int \frac{\cancel{A}\,3}{x} + \frac{\cancel{B}\,-1}{x^2} + \frac{\cancel{C}\,1}{x+1}\,dx$$

$$= \frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$$

$$= \frac{(A+C)x^2+(A+B)x+B}{x^2(x+1)}$$

$A+C=4$

$A+B=2$

$B=-1$

$A-1=2$

$A=3$

$C=1$