Textbook Problems:

- 3.9 #5, 9; 11-19 odd; **37**, 45, ~~49~~
- 4.1 #**3-31** odd; **35-41 odd**; **53,** 67, **75**, ~~55-61 odd; 83~~
- 4.2 #7-**23**odd; **33-41odd,45,51**, ~~27-37odd; 41,43,47,53~~
- 4.3 #7,17,37,43,~~45~~,**47**
- 4.4 #13, 15, 23, ~~31~~, **33**

$$\int (\tan^2 y + 1)\,dy$$
$$= \int \sec^2 y\,dy$$
$$= \boxed{\tan y + C}$$

$$\frac{\sin^2 x + \cos^2 x = 1}{\cos^2 x}$$
$$\tan^2 x + 1 = \sec^2 x$$

$f(x) = \sin x \qquad (2, \sin 2) \qquad \pi \approx 3.14$

$\frac{\pi}{2} \approx 1.55$

$c = \pi/2 \quad f(c) = \sin\frac{\pi}{2} = 1 \ ; \ f'(x) = \cos x \ ; \ f'(\frac{\pi}{2}) = 0$

$T(x) = f(c) + f'(c)\Delta x$

$T(x) = 1 + 0 \cdot (x - \frac{\pi}{2}) = 1$

$f(2) = f(\frac{\pi}{2} + \Delta x)$

$2 - \frac{\pi}{2}$

$f(x) = \sin x \qquad (2, \sin 2)$

$f'(x) = \cos x$

$m = f'(2) = \cos 2$

$y - \sin 2 = \cos 2(x - 2)$

$y = (\cos 2)x - 2\cos 2 + \sin 2$

4.2 Area

Sigma Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

Summation Formulas

1. $\sum_{i=1}^{n} c = nc$

2. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$

Use sigma notation to write the sum.

8. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\dots+\frac{5}{1+15}$

$$=\sum_{i=1}^{15}\frac{5}{1+i}$$

14. $\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{0}{n}\right)^2}+\dots+\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{n-1}{n}\right)^2}$

$$=\sum_{i=0}^{n-1}\frac{1}{n}\sqrt{1-\left(\frac{i}{n}\right)^2}=\sum_{i=1}^{n}\frac{1}{n}\sqrt{1-\left(\frac{i-1}{n}\right)^2}$$

Use summation properties to evaluate the sum.

20. $\sum_{i=1}^{10} i(i^2+1)=\sum_{i=1}^{10}(i^3+i)$

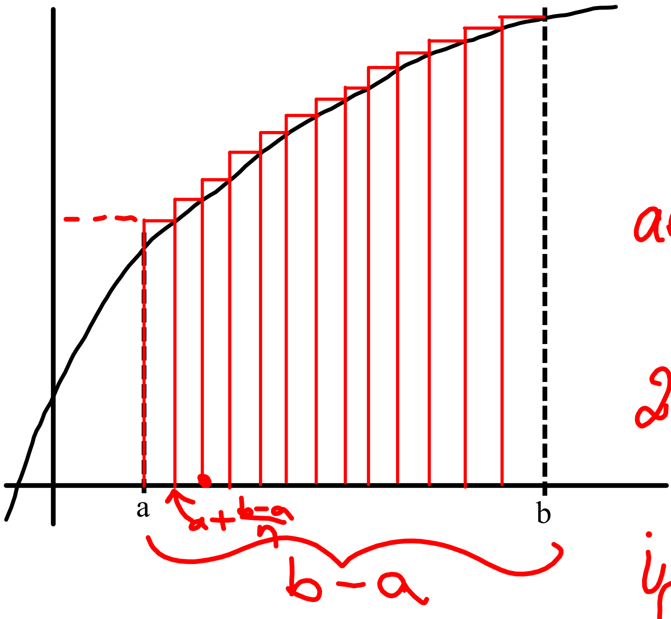
$$=\sum_{i=1}^{10} i^3+\sum_{i=1}^{10} i$$

$$=\frac{10^2(10+1)^2}{4}+\frac{10(10+1)}{2}$$

$$=\frac{\cancel{100}^{25}(121)}{\cancel{4}}+5(11)$$

$$=25(121)+55=\boxed{3080}$$

4.2 Area



Divide the interval [a,b] into n equal subintervals, each of width (b-a)/n.

Here, the height of a rectangle is determined by the right endpoint of each subinterval; this is called an upper sum.

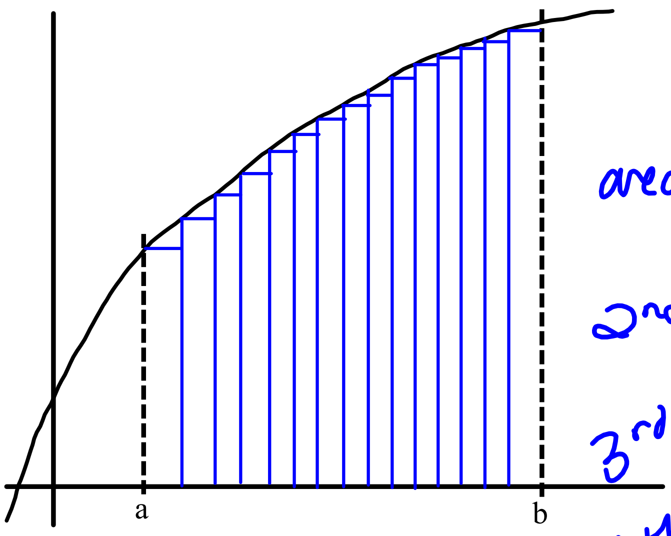
area of 1^{st} rectangle:

$$\left(\frac{b-a}{n}\right)\cdot f\left(a+\frac{b-a}{n}\right)$$

2^{nd}: $\left(\frac{b-a}{n}\right)\cdot f\left(a+2\cdot\frac{b-a}{n}\right)$

i^{th}: $\left(\frac{b-a}{n}\right) f\left(a+i\frac{b-a}{n}\right)$

$$\text{Total Area} = \boxed{\sum_{i=1}^{n}\left(\frac{b-a}{n}\right) f\left(a+i\,\frac{b-a}{n}\right)}$$



Here, the height of a rectangle is determined by the left endpoint of each subinterval; this is called a lower sum.

area of 1^{st}:

$$\left(\frac{b-a}{n}\right)\cdot f\left(a+0\cdot\frac{b-a}{n}\right)$$

2^{nd}: $\left(\frac{b-a}{n}\right)\cdot f\left(a+1\cdot\frac{b-a}{n}\right)$

3^{rd}: $\left(\frac{b-a}{n}\right)\cdot f\left(a+2\cdot\frac{b-a}{n}\right)$

i^{th}: $\left(\frac{b-a}{n}\right) f\left(a+(i-1)\frac{b-a}{n}\right)$

$$\text{Total Area}: \sum_{i=1}^{n}\left(\frac{b-a}{n}\right)\cdot f\left(a+(i-1)\frac{b-a}{n}\right)$$

Lower sum: $s(n) = \sum_{i=1}^{n} f(m_i) \Delta x$

Upper sum: $S(n) = \sum_{i=1}^{n} f(M_i) \Delta x$

$f(m_i)$ = minimum function value in an interval
$f(M_i)$ = Maximum function value in an interval
$\Delta x = \frac{b-a}{n}$ $s(n) \leq S(n)$

Area of the region bounded by the graph of f, the x-axis, & the lines x=a & x=b is

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \Delta x, \quad x_{i-1} \leq c_i \leq x_i$$

where $\Delta x = \frac{b-a}{n}$.