Textbook Problems:

- 3.9 #5, 9; 11-19 odd; **37**, 45, ~~49~~
- 4.1 #**3-31** odd; **35-41 odd**; **53,** 67, **75**, ~~55-61 odd; 83~~
- 4.2 #7-**23**odd; **33-41odd,45,51**, ~~27-37odd; 41,43,47,53~~
- 4.3 #7,17,37,43,~~45~~,**47**
- 4.4 #13, 15, 23, ~~31~~, **33**

Expect Quiz Wed. 11/16

Test Wed. 11/30?

4.3 Riemann Sums & Definite Integrals

$$\sum_{i=1}^{n} f(c_i)\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i ,$$

where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \dots < x_n = b$ is called the Riemann Sum of f.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i) \cdot \frac{b-a}{n} =$$

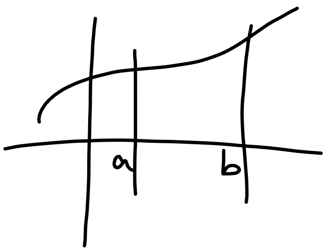
$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x = \int_a^b f(x)\,dx$$

called the definite integral of f from a to b.

Properties

If $f(a)$ is defined,

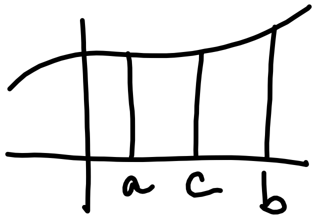
$$\int_a^a f(x)\,dx = 0$$



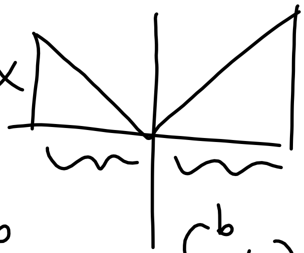
If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



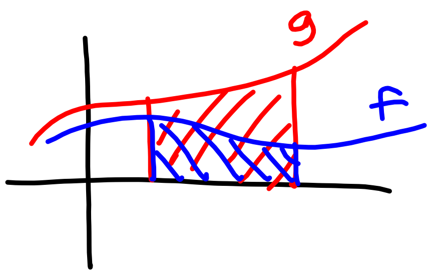
$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$ on $[a, b]$

$$\int_a^b f(x)\,dx \geq 0$$



If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

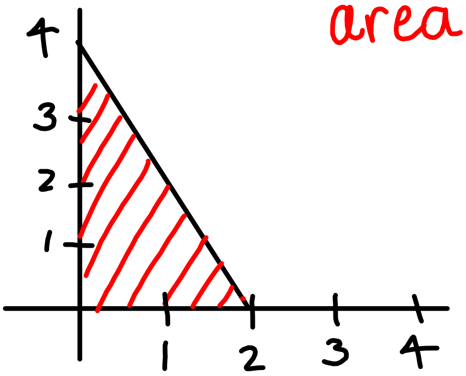
8. $\int_{-1}^{2}(3x^2+2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(3\left(-1+\frac{3}{n}i\right)^2+2\right)\cdot\left(\frac{2-(-1)}{n}\right)$

height of i^{th} interval

width of each interval

$= \lim_{\Delta x\to 0}\sum_{i=1}^{n}\left[3(-1+\Delta x\, i)^2+2\right]\cdot\Delta x$

14. $f(x)=4-2x$



area $= \int_{0}^{2}(4-2x)\,dx$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[4-2\left(\frac{2i}{n}\right)\right]\cdot\frac{2}{n}$

$= \boxed{4}$

40. $\int_2^4 x^3 dx = 60$; $\int_2^4 x\,dx = 6$; $\int_2^4 dx = 2$

$$\int_2^4 (6 + 2x - x^3)\,dx$$

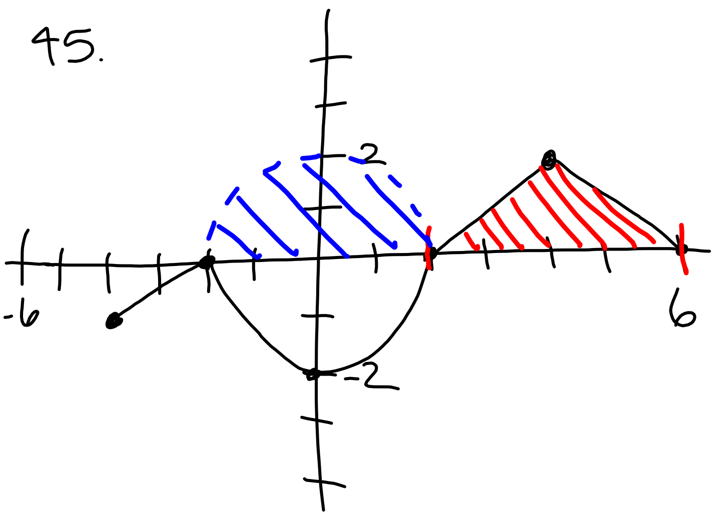
$$= \int_2^4 6\,dx + \int_2^4 2x\,dx + \int_2^4 -x^3 dx$$

$$= 6\int_2^4 dx + 2\int_2^4 x\,dx - \int_2^4 x^3 dx$$

$$= 6(2) + 2(6) - 60$$

$$= 12 + 12 - 60 = \boxed{-36}$$

45.



b) $\int_2^6 f(x)\,dx$

$= \boxed{4}$

$\int_{-2}^{2} |f(x)|\,dx$

$= \frac{1}{2}\pi(2)^2 = \boxed{2\pi}$

4.4 The Fundamental Theorem of Calculus

If f is continuous on [a,b] and F is the antiderivative of f on [a,b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

10. $\int_1^3 (3x^2 + 5x - 4)\,dx$

$$= x^3 + \frac{5}{2}x^2 - 4x \Big|_{x=1}^{3}$$

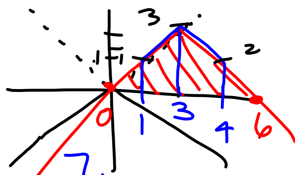
$$= 3^3 + \frac{5}{2}(3)^2 - 4(3) - \left[1^3 + \frac{5}{2}(1)^2 - 4(1)\right]$$

$$= 27 + \frac{45}{2} - 12 - 1 - \frac{5}{2} + 4$$

$$= 20 + 27 - 12 - 1 + 4$$

$$= 38$$

24. $\int_1^4 (3-|x-3|)\,dx$



$$= \int_1^3 \left[3-(-(x-3))\right]dx + \int_3^4 \left[3-(x-3)\right]dx$$

$$= \int_1^3 x\,dx + \int_3^4 (-x+6)\,dx$$

$$= \underbrace{\tfrac{1}{2}(1+3)(2)}_{4} + \underbrace{\tfrac{1}{2}(3+2)(1)}_{5/2} \leftarrow$$ adding areas of 2 trapezoids

$$= \tfrac{1}{2}x^2\Big|_1^3 + \left(-\tfrac{1}{2}x^2+6x\right)\Big|_3^4$$

$$= \tfrac{1}{2}(3)^2 - \tfrac{1}{2}(1)^2 + \left[-\tfrac{1}{2}(4)^2+6(4)\right] - \left[-\tfrac{1}{2}(3)^2+6(3)\right]$$

$$= \frac{9}{\cancel{2}} - \frac{1}{2} - 8 + 24 + \frac{9}{\cancel{2}} - 18$$

$$= 9 - \tfrac{1}{2} + 16 - 18$$

$$= 7 - \tfrac{1}{2} = \boxed{\tfrac{13}{2}}$$

32. $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\,dt$

$$= t^2 + \sin t\,\Big|_{-\pi/2}^{\pi/2}$$

$$= \left(\frac{\pi}{2}\right)^2 + \sin\frac{\pi}{2} - \left[\left(\frac{-\pi}{2}\right)^2 + \sin\left(\frac{-\pi}{2}\right)\right]$$

$$= \frac{\pi^2}{4} + 1 - \frac{\pi^2}{4} + 1 = \boxed{2}$$

4.3

44. Given $\int_{-1}^{1} f(x)\,dx = 0$ & $\int_{0}^{1} f(x)\,dx = 5$

$\int_a^c = \int_a^b + \int_b^c$

$\int_a^b = -\int_b^a$

(a) $\int_{-1}^{0} f(x)\,dx = \boxed{-5}$

$\int_{-1}^{1} = \int_{-1}^{0} + \int_{0}^{1}$

(b) $\int_0^1 f(x)\,dx - \int_{-1}^{0} f(x)\,dx$

$5 - (-5) = \boxed{10}$

(c) $\int_{-1}^{1} 3f(x)\,dx = 3 \cdot 0 = \boxed{0}$

(d) $\int_0^1 3f(x)\,dx$

$= 3 \cdot 5 = \boxed{15}$

(e) $\int_1^0 f(x)\,dx =$

$\int_1^0 = -\int_0^1 = \boxed{-5}$