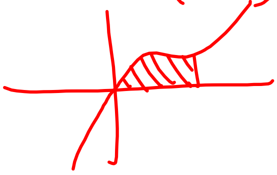
4.4 #45-55 odd; 75-91 odd
4.5 #5-25 odd; 33-61 odd
5.2 #1-37 odd; 49-55 odd; 63, 65
5.4 #91-117 odd
5.5 #71-82 all
5.7 #1-45 odd
Ch 5 Review pp.393-394 #15-22, 47-54,65-66, 77-82

TEST #2 - Thurs. 12/15

1. Find the area of the region bounded by the graphs of the equations $y = x^3 + x$, $x = 4$, $y = 0$. Round your answer to the nearest whole number.

   a. 768
   b. 96
   c. 49
   d. 72
   e. 16

$x(x^2+1)$



$$\int_0^4 (x^3+x)\,dx$$

$$= \frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_0^4$$

$$= \frac{1}{4}(4)^4 + \frac{1}{2}(4)^2 - 0$$

2. Find the average value of the function over the given interval and all values t in the interval for which the function equals its average value.

$f(t) = \frac{t^2+4}{t^2}$, $1 \le t \le 5$

Use a graphing utility to verify your results.

a. The average is $\frac{9}{5}$ and the point at which the function is equal to its mean value is $\sqrt{5}$.

b. The average is $\frac{9}{5}$ and the point at which the function is equal to its mean value is $\sqrt{5}$ and $-\sqrt{5}$.

c. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $-\sqrt{5}$.

d. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $\sqrt{5}$.

e. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $\sqrt{5}$ and $-\sqrt{5}$.

$$\frac{1}{5-1}\int_1^5 \frac{t^2+4}{t^2}\,dt = \frac{1}{4}\int_1^5 \left(1+4t^{-2}\right)dt$$

$$= \frac{1}{4}\left(t - \frac{4}{t}\right)\Big|_1^5 =$$

$$\frac{1}{4}\left(5-\frac{4}{5}\right) - \frac{1}{4}\left(1-\frac{4}{1}\right)$$

$$= \boxed{\frac{9}{5}}$$

$$\frac{9}{5} = \frac{t^2+4}{t^2}$$

$$9t^2 = 5t^2 + 20$$

$$4t^2 = 20$$

$$t^2 = 5 \qquad t = \pm\sqrt{5}$$

3. Determine all values of x in the interval $[1,3]$ for which the function $f(x) = \frac{4\left(x^2+1\right)}{x^2}$ equals its average value $\frac{16}{3}$.

a. $x = \sqrt{7}$
b. $x = 4$
c. $x = 1$
d. $x = \pm\sqrt{2}$
e. $x = \sqrt{3}$

$$\frac{4(x^2+1)}{x^2} = \frac{16}{3}$$

$$12(x^2+1) = 16x^2$$

$$3(x^2+1) = 4x^2$$

$$3x^2+3 = 4x^2$$

$$3 = x^2$$

$$x = \pm\sqrt{3}$$

4. Find $F'(x)$ given

$$F(x) = \int_{-2x}^{2x} s^2\,ds.$$

a. $F'(x) = 16x^2$
b. $F'(x) = 4x^2$
c. $F'(x) = 8x^2$
d. $F'(x) = 0$
e. $F'(x) = 24x^2$

$$\int_{-2x}^{2x} = \int_{-2x}^{a} + \int_{a}^{2x} = -\int_{a}^{-2x} + \int_{a}^{2x}$$

$$F(x) = -\int_{a}^{-2x} s^2\,ds + \int_{a}^{2x} s^2\,ds$$

$$F'(x) = -(-2x)^2\cdot(-2) + (2x)^2\cdot 2$$

$$= 8x^2 + 8x^2 = 16x^2$$

5. Find the indefinite integral $\int t^4\left(4+t^5\right)^3 dt$.

a. $\dfrac{\left(4+t^5\right)^4}{4} + C$
b. $\dfrac{\left(4+t^5\right)^4}{20} + C$
c. $20\left(4+t^5\right)^4 + C$
d. $\dfrac{\left(4+t^4\right)^4}{20} + C$
e. $\dfrac{\left(4+t^5\right)^4}{10} + C$

$u = 4 + t^5$

$du = 5t^4\,dt$

$\frac{1}{5}du = t^4\,dt$

$$\int \frac{1}{5}u^3\,du = \frac{1}{20}u^4 + C$$

$$= \frac{1}{20}\left(4+t^5\right) + C$$

6. Find the indefinite integral of the following function and check the result by differentiation.

$$\int \frac{2s}{\left(s^2+5\right)^3}\,ds$$

$u = s^2+5$

$du = 2s\,ds$

$$\int \frac{du}{u^3} = \int u^{-3}\,du$$

$$= -\frac{1}{2}u^{-2} + C$$

$$= -\frac{1}{2}\cdot\frac{1}{\left(s^2+5\right)^2} + C$$

a. $\boxed{\dfrac{-1}{2\left(s^2+5\right)^2}+C}$

b. $\dfrac{1}{2\left(s^2+5\right)^2}+C$

c. $\dfrac{-1}{2\left(s^2+5\right)^4}+C$

d. $\dfrac{-1}{2\left(s^2+5\right)^3}+C$

e. $\dfrac{-1}{2\left(s^2+5\right)^{-2}}+C$

7. Find the indefinite integral of the following function and check the result by differentiation.

$$\int (7-z)\sqrt{z}\,dz = \int (7-z)\cdot z^{1/2}\,dz = \int \left(7z^{1/2} - z^{3/2}\right)dz$$

$$= \frac{14}{3}z^{3/2} - \frac{2}{5}z^{5/2} + C$$

a. $\frac{14}{3}z^{\frac{3}{2}} - \frac{5}{2}z^{\frac{5}{2}} + C$

b. $\frac{14}{3}z^{\frac{3}{2}} + \frac{5}{2}z^{\frac{5}{2}} + C$

c. $\boxed{\frac{14}{3}z^{\frac{3}{2}} - \frac{2}{5}z^{\frac{5}{2}} + C}$

d. $\frac{7}{3}z^{\frac{3}{2}} - \frac{5}{2}z^{\frac{5}{2}} + C$

e. $\frac{7}{3}z^{\frac{3}{2}} - \frac{2}{5}z^{\frac{5}{2}} + C$

8. Find the indefinite integral of the following function.

$\int \frac{\sin u}{\cos^3 u} du$

a. $\frac{(\cos u)^{-2}}{2} + C$

b. $\frac{(\sin u)^{-2}}{3} + C$

c. $\frac{(\cos u)^{-3}}{2} + C$

d. $\frac{(\cos u)^{-2}}{3} + C$

e. $\frac{(\sin u)^{-2}}{2} + C$

$x = \cos u$

$dx = -\sin u\, du$

$-dx = \sin u\, du$

$\int -x^{-3} dx$

$= \frac{1}{2} x^{-2} + c$

$= \frac{1}{2\cos^2 x} dx$

$\int \tan u \sec^2 u\, du$

$x = \tan u$

$dx = \sec^2 u\, du$

$= \int x\, dx$

$\frac{1}{2} x^2 + C$

$\frac{1}{2} \tan^2 x + c$

$\tan^2 x + 1 = \sec^2 y$

9. Find the indefinite integral $\int \frac{x^2}{4x^3 + 9} dx$.

a. $\frac{x^3}{4x^4 + 9x} + C$

b. $\ln|4x^3 + 9| + C$

c. $\frac{1}{4} \ln|4x^3 + 9| + C$

d. $\frac{1}{12} \ln|4x^3 + 9| + C$

e. $12 \ln|4x^3 + 9| + C$

$u = 4x^3 + 9$

$du = 12x^2 dx$

$\frac{1}{12} du = x^2 dx$

$\int \frac{1}{12} \frac{du}{u} = \frac{1}{12} \ln|4x^3 + 9| + c$

10. Find the indefinite integral $\int \frac{(\ln x)^6}{x}\,dx$.

a. $\frac{(\ln x)^7}{6}+C$

b. $\frac{(\ln x)^7}{7}+C$

c. $\frac{(\ln x)^6}{6}+C$

d. $6(\ln x)^5+C$

e. $\frac{7\ln x}{x}+C$

$u=\ln x$

$du=\frac{dx}{x}$

$\int u^6\,du$

$\frac{1}{7}u^7+C$

$\frac{1}{7}(\ln x)^7+C$

11. Find $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx.$ $=\int \frac{1}{12}\cdot\frac{dx}{x\ln x}=\int\frac{1}{12}\frac{du}{u}=\boxed{\frac{1}{12}\ln\left|\ln x\right|+C}$

a. $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx=\frac{1}{12}\ln\left|\ln\left(x^{12}\right)\right|+C$

b. $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx=\ln\left|x^{12}\right|+C$

c. $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx=12\ln\left|\ln\left(x^{12}\right)\right|+C$

d. $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx=12\ln\left|x^{12}\right|+C$

e. $\int \frac{1}{x\ln\left(x^{12}\right)}\,dx=\frac{1}{12}\ln\left|x^{12}\right|+C$

$u=\ln(x^{12})$

$\frac{1}{12}\cdot du=\frac{1}{x^{12}}\cdot 12x^{11}dx=\frac{12}{x}dx\cdot\frac{1}{12}$

$\int\frac{1}{12}\cdot\frac{du}{u}=\frac{1}{12}\ln\left|\underbrace{\ln(x^{12})}_{12\ln x}\right|+C$

$\ln 12+\ln|\ln x|$

12. Find $\int \tan 6\theta \, d\theta$.

a. $\int \tan 6\theta \, d\theta = -\frac{1}{6}\ln|\sec 6\theta| + C$

b. $\int \tan 6\theta \, d\theta = \frac{1}{6}\ln|\cos 6\theta| + C$

c. $\int \tan 6\theta \, d\theta = -6\ln|\cos 6\theta| + C$

d. $\int \tan 6\theta \, d\theta = -6\ln|\sec 6\theta| + C$

e. $\int \tan 6\theta \, d\theta = -\frac{1}{6}\ln|\cos 6\theta| + C$

$\int \frac{\sin 6\theta}{\cos 6\theta} d\theta = \int -\frac{1}{6}\frac{du}{u}$

$u = \cos 6\theta$

$du = -\sin 6\theta \cdot 6 \, d\theta$

$-\frac{1}{6} du = \sin 6\theta \, d\theta$

$-\frac{1}{6}\ln|\cos 6\theta| + C$

13. Evaluate $\int_1^e \frac{(1+\ln x)^5}{x} dx$.

a. $\int_1^e \frac{(1+\ln x)^5}{x} dx = \frac{31}{5}$

b. $\int_1^e \frac{(1+\ln x)^5}{x} dx = \frac{31}{7}$

c. $\int_1^e \frac{(1+\ln x)^5}{x} dx = \frac{21}{2}$

d. $\int_1^e \frac{(1+\ln x)^5}{x} dx = \frac{2}{7}$

e. $\int_1^e \frac{(1+\ln x)^5}{x} dx = \frac{32}{3}$

$u = 1 + \ln x$

$du = \frac{dx}{x}$

$\int_{x=1}^{e} u^5 \, du = \frac{1}{6}u^6 \Big|_{x=1}^{e}$

$\frac{1}{6}(1+\ln x)^6 \Big|_1^e$

$\frac{1}{6}(1+\underbrace{\ln e}_{1})^6 - \frac{1}{6}(1+\underbrace{\ln 1}_{0})^6$

$\frac{1}{6}(2)^6 - \frac{1}{6}(1)^6$

14. Find $F'(x)$ given $F(x) = \int_1^{6x} \frac{7}{t}\,dt$.

$= \frac{7}{6x} \cdot 6 = \frac{7}{6}$

a. $F'(x) = \frac{13}{x}$
b. $F'(x) = \ln|7x|$
c. $F'(x) = \frac{6}{x}$
d. $F'(x) = \ln|6x|$
e. $F'(x) = \frac{7}{x}$

15. Find the indefinite integral.

$\int 2xe^{4x^2}\,dx$

$u = 4x^2$
$du = 8x\,dx$
$\frac{1}{4}du = 2x\,dx$

$\int \frac{1}{4} e^u\,du$
$= \frac{1}{4}e^u + C$
$= \frac{1}{4}e^{4x^2} + C$

a. $16x^2e^{4x^2} + C$
b. $\frac{1}{4}xe^{4x^2} + C$
c. $\frac{1}{4}e^{4x^2} + C$
d. $\frac{1}{2}e^{4x^2} + C$
e. $8xe^{4x^2} + C$

16. Evaluate the definite integral.

$$\int_{\ln 2}^{\ln 4} e^{-x}\,dx$$

a. 6
b. $\frac{1}{16}$
c. $\frac{3}{4}$
d. $\frac{1}{4}$
e. 2

$u=-x$
$du=-dx$
$-du=dx$

$$\int_{x=\ln 2}^{\ln x} -e^u\,du = -e^u\Big|_{x=\ln 2}^{\ln 4}$$

$$-e^{-x}\Big|_{\ln 2}^{\ln 4}$$

$\log_a(b^x) = x\log_a b$

$$= -e^{-\ln 4} - \left(-e^{-\ln 2}\right)$$

$$= -e^{\ln \frac{1}{4}} + e^{\ln \frac{1}{2}}$$

$$= -\frac{1}{4} + \frac{1}{2}$$

17. Find the following indefinite integral.

$$\int x^7 \left(4^{-x^8}\right) dx$$

a. $\frac{-4^{-x^8}}{8\ln(4)}+C$
b. $\frac{-4^{-x^8}}{4\ln(8)}+C$
c. $\frac{-4^{-x^8+1}}{x^{8+1}}+C$
d. $\frac{-4^{-x^8}}{\ln(4)}+C$
e. $\frac{4^{-x^8}}{\ln(4)}+C$

$u=-x^8$
$du=-8x^7\,dx$
$-\frac{1}{8}du=x^7\,dx$

$$\int -\frac{1}{8}4^u\,du$$

$$= -\frac{1}{8}\cdot\frac{1}{\ln 4}4^{-x^8}+c$$

18. Find the indefinite integral.

$$\int \frac{1}{x\sqrt{64x^2-49}}\,dx = \int \frac{1}{x\sqrt{(8x)^2-7^2}}\,dx \cdot \frac{8}{8}$$

a. $\frac{1}{7}\operatorname{arcsec}\left(\frac{|8|x}{7}\right)+C$

b. $7\operatorname{arcsec}\left(\frac{|7|x}{8}\right)+C$

c. $\frac{1}{7}\operatorname{arcsec}\left(\frac{|64|x}{49}\right)+C$

d. $\frac{1}{7}\operatorname{arcsec}\left(\frac{|7|x}{8}\right)+C$

e. $7\operatorname{arcsec}\left(\frac{|64|x}{49}\right)+C$

$$= \int \frac{du}{u\sqrt{u^2-7^2}}$$

$$= \frac{1}{7}\operatorname{arcsec}\frac{|8x|}{7} + C$$

19. Find the integral $\int \frac{x-15}{x^2+1}\,dx$.

a. $15\ln\left(x^2+1\right)-\arctan(x)+C$

b. $\frac{1}{15\ln\left(x^2+1\right)}-15\arctan(x)+C$

c. $\frac{1}{2\ln\left(x^2+1\right)}+15\arctan(x)+C$

d. $\frac{1}{2}\ln\left(x^2+1\right)-15\arctan(x)+C$

e. $2\ln\left(x^2+1\right)-15\arctan(x)+C$

$$= \int \frac{x\,dx}{x^2+1} - \int \frac{15\,dx}{x^2+1}$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{1}{2}\frac{du}{u} - 15\arctan x + c$$

>0

$$\frac{1}{2}\ln\left|x^2+1\right| - 15\arctan x + c$$

20. Evaluate the integral $\int_0^{1/6} \frac{16}{\sqrt{1-9x^2}}\,dx$. $= \int_0^{1/6} \frac{16\,dx}{\sqrt{1^2-(3x)^2}}$

$u=3x$
$du=3dx$
$\frac{1}{3}du=dx$

a. $\frac{16}{25\pi}$
b. $\frac{8}{9}\pi$
c. $\frac{4}{9\pi}$
d. $\frac{16}{25}\pi$
e. $\frac{8}{9\pi}$

$= \frac{16}{3}\arcsin 3x \Big|_0^{1/6}$

$= \frac{16}{3}\arcsin\frac{1}{2} - \frac{16}{3}\arcsin 0 \to 0$

$= \frac{16}{3}\cdot\frac{\pi}{6}$

21. Find the indefinite integral.

$\int \frac{2x-4}{x^2+4x+13}\,dx = \int \frac{2x+4-8}{x^2+4x+13}\,dx = \int \frac{2x+4}{x^2+4x+13}\,dx - \int \frac{8\,dx}{(x+2)^2+3^2}$

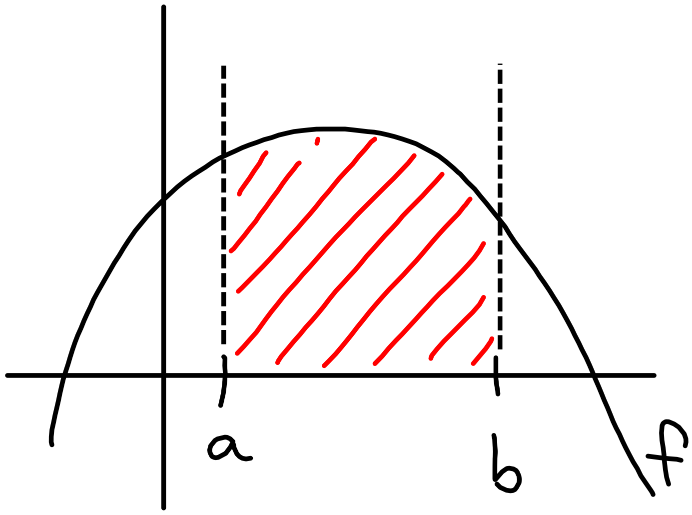
$x^2+4x+4 \underbrace{-4+13}_{9}$

a. $\ln|x^2+4x+13| - \frac{8}{3}\arctan\left(\frac{x+2}{3}\right)+C$
b. $\ln|x^2+4x+13|+C$
c. $-\frac{8}{3}\arctan\left(\frac{x+2}{3}\right)+C$
d. $\ln|x^2+4x+13|+\frac{8}{3}\arctan\left(\frac{x+2}{3}\right)+C$
e. $\frac{8}{3}\arctan\left(\frac{x+2}{3}\right)+C$

$= \int \frac{du}{u} - 8\arctan\frac{(x+2)}{3}$

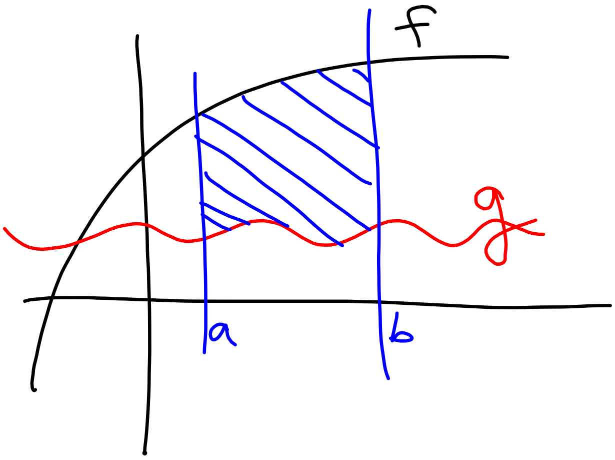
$= \ln|x^2+4x+13| - 8\arctan\frac{x+2}{3} + C$

6.1 - Area Between Curves



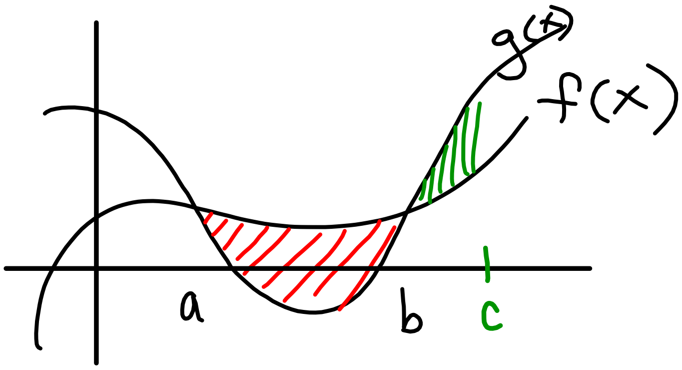
Area of region bounded by f(x) & x-axis, between a and b is

$$\int_a^b f(x)\,dx$$



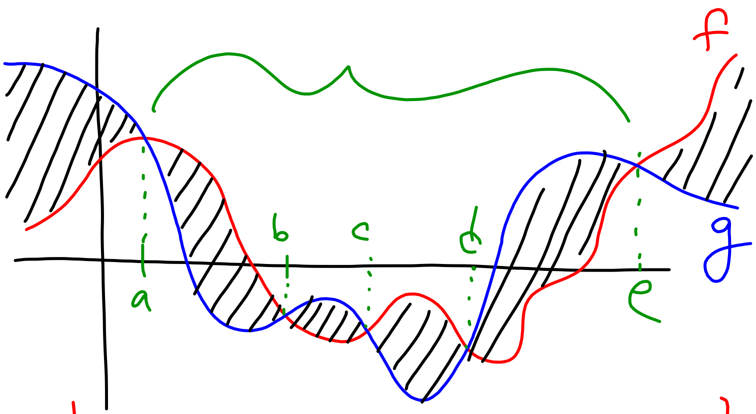
$$\int_a^b \left(f(x)-g(x)\right)dx$$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$



$$\int_a^b \left(f(x) - g(x)\right) dx$$

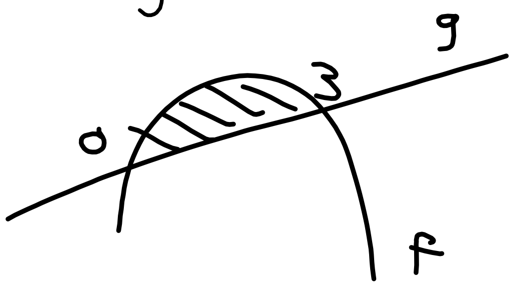
$$\int_b^c \left(g(x) - f(x)\right) dx$$



$$\int_a^b (f-g)\,dx + \int_b^c (g-f)\,dx + \int_c^d (f-g)\,dx + \int_d^e (g-f)\,dx$$

6.1 Find the area between the curves.

#18. $f(x) = -x^2 + 4x + 1$
$g(x) = x + 1$



$-x^2 + 4x + 1 = x + 1$
$0 = x^2 - 3x$
$0 = x(x-3)$
$x = 0, 3$

$$\int_0^3 \left[\left(-x^2+4x+1\right) - (x+1)\right] dx$$
$$= \int_0^3 \left(-x^2+3x\right) dx = -\frac{1}{3}x^3 + \frac{3}{2}x^2 \Big|_0^3$$
$$= -9 + \frac{27}{2} = \boxed{\frac{9}{2}}$$