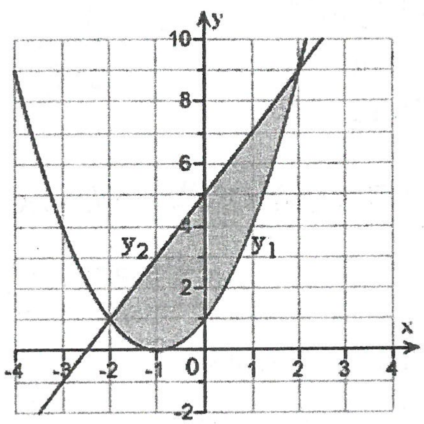
Name: KEY Class: ______________ Date: __________ ID: A

# Integral Calculus - Quiz #4

## Multiple Choice

*Identify the choice that best completes the statement or answers the question.*

A 1. Set up the definite integral that gives the area of the region bounded by the graph of $y_1 = x^2 + 2x + 1$ and $y_2 = 2x + 5$.




$x^2 + 2x + 1 = 2x + 5$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2, 2$

$\int_{-2}^{2} [(2x+5) - (x^2 + 2x + 1)]\,dx$

$= \int_{-2}^{2} (-x^2 + 4)\,dx$

a. $\boxed{\int_{-2}^{2} \left(-x^2 + 4\right) dx}$

b. $\int_{-2}^{2} \left(x^2 + 2x + 1\right) dx$

c. $\int_{-2}^{2} \left(-x^2 + 4\right) dy$

d. $\int_{-2}^{2} \left(x^2 + 4x + 6\right) dy$

e. $\int_{-2}^{2} \left(x^2 + 4x + 6\right) dx$

Name: ______________________ ID: A

A

2. Find the area of the region bounded by the graphs of the algebraic functions.

$f(x) = \sqrt[3]{x-8}$

$g(x) = x - 8$

a. $A = \dfrac{1}{2}$

b. $A = \dfrac{1}{17}$

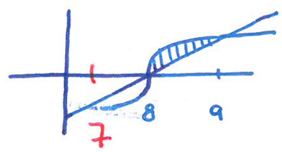
c. $A = \dfrac{1}{24}$

d. $A = \dfrac{23}{24}$

e. $A = \dfrac{15}{16}$

$\sqrt[3]{x-8} = x-8$

$x = 8, 9$




$\int_8^9 \left(\sqrt[3]{x-8} - (x-8)\right)dx$

$= \int_8^9 \sqrt[3]{x-8}\,dx - \int_8^9 (x-8)\,dx$

$u = x-8$

$du = dx$

$\int u^{1/3}du = \frac{3}{4}u^{4/3}$

$= \frac{3}{4}(x-8)^{4/3}\Big|_8^9 - \left(\frac{x^2}{2} - 8x\right)\Big|_8^9$

$= \left(\frac{3}{4} - 0\right) - \left[\left(\frac{81}{2} - 72\right) - \left(\frac{64}{2} - 64\right)\right] = \frac{3}{4} - \frac{17}{2} + 8$

$= \frac{3}{4} - \frac{34}{4} + \frac{32}{4} = \boxed{\frac{1}{4}}$

* use calculator to solve $\sqrt[3]{x-8} = x-8$

* correct answer choice is not listed!

$+ \int_7^8 \left((x-8) - \sqrt[3]{x-8}\right)dx = \frac{1}{4}$

$+$

D

3. Find the area of the region bounded by the graphs of the equations.

$f(x) = \sin(x)$, $g(x) = \cos(2x)$, $\dfrac{-\pi}{2} \le x \le \dfrac{\pi}{6}$.

a. $A = \dfrac{9}{2}$

b. $A = \dfrac{9}{8}$

c. $A = \dfrac{3}{8}$

d. $A = \dfrac{3^{3/2}}{4}$

e. $A = \dfrac{3}{2}$

$\sin x = \cos 2x$

$\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$, $\sin x = -1$

$x = \frac{\pi}{6}$ $\qquad x = -\frac{\pi}{2}$

* graphs intersect only at endpoints of intervals

$\int_{-\pi/2}^{\pi/6} (\cos 2x - \sin x)\,dx$

$= \frac{1}{2}\sin 2x + \cos x \Big|_{-\pi/2}^{\pi/6}$

$= \left(\frac{1}{2}\sin\frac{\pi}{3} + \cos\frac{\pi}{6}\right) - \left(\frac{1}{2}\sin(-\pi) + \cos\left(-\frac{\pi}{2}\right)\right)$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\cdot\frac{2}{2} - 0 - 0$

$= \frac{3\sqrt{3}}{4}$

A 4. Set up and evaluate the integral that gives the volume of the solid formed by revolving the region bounded by $y = x^{\frac{3}{4}}$, $y = 1$, and $x = 0$ about the $y$-axis.

$y^{4/3} = x$

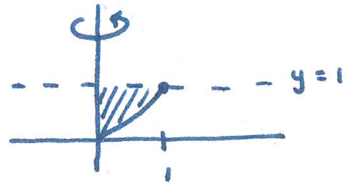
a. $\boxed{V = \pi \int_0^1 y^{\frac{8}{3}}\, dy = \frac{3}{11}\pi}$

b. $V = \pi \int_0^1 y^{\frac{8}{3}}\, dy = \frac{3}{22}\pi$

c. $V = \pi \int_0^1 y^{\frac{4}{3}}\, dy = \frac{3}{11}\pi$

d. $V = \pi \int_0^1 y^{\frac{3}{4}}\, dy = \frac{3}{22}\pi$

e. $V = \pi \int_0^1 y^{\frac{3}{8}}\, dy = \frac{3}{11}\pi$




$\int_0^1 \pi \left(y^{4/3}\right)^2 dy$

$= \int_0^1 \pi y^{8/3}\, dy =$

$= \frac{3\pi}{11} y^{11/3} \Big|_0^1 = \frac{3\pi}{11}$

A 5. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = 2x^2$, $y = 0$, and $x = 2$ about the line $x = 2$.

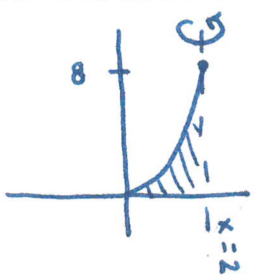
a. $\boxed{\frac{16}{3}\pi}$

b. $\frac{8}{3}\pi$

c. $\frac{16}{3}$

d. $\frac{32}{3}$

e. $\frac{32}{3}\pi$




$\frac{y}{2} = x^2$

$\sqrt{\frac{y}{2}} = x$

$\int_0^8 \pi \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$

$= \int_0^8 \pi \left(4 - 4\sqrt{\frac{y}{2}}) + \frac{y}{2}\right) dy$

$= 4\pi y - 2\sqrt{2} \cdot \frac{2}{3} y^{3/2} \pi + \frac{1}{4} y^2 \pi \Big|_0^8$

$= 4\pi y - \frac{4\sqrt{2}}{3}\pi y^{3/2} + \frac{\pi}{4} y^2 \Big|_0^8$

$= 32\pi - \frac{4\sqrt{2}}{3}\pi \cdot (2\sqrt{2})^3 + \frac{\pi}{4} \cdot 64$

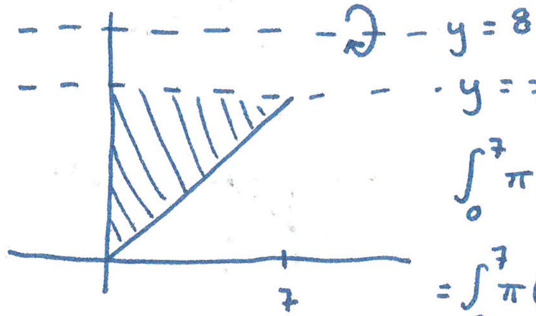
$= 48\pi - \frac{128\pi}{3}$

$= \frac{144\pi}{3} - \frac{128\pi}{3} = \frac{16\pi}{3}$

Name: ______________________ ID: A

$\underline{\text{D}}$ 6. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 8$.

$y = x,\ y = 7,\ x = 0$




a. $\frac{245}{3}\pi$

b. $\frac{637}{6}\pi$

c. $\frac{637}{3}\pi$

$\boxed{\text{d. } \frac{490}{3}\pi}$

e. $\pi$

$$\int_0^7 \pi(8-x)^2\,dx - \pi(1)^2 \cdot 7$$

$$= \int_0^7 \pi(64-16x+x^2)\,dx - 7\pi$$

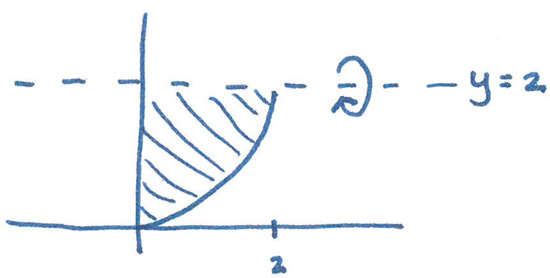
$$= 64\pi x - 8\pi x^2 + \frac{\pi}{3}x^3 \Big|_0^7 - 7\pi$$

$$= 448\pi - 392\pi + \frac{343\pi}{3} - 7\pi$$

$$= 49\pi + \frac{343\pi}{3} = \frac{147\pi}{3} + \frac{343\pi}{3} = \frac{490\pi}{3}$$

$\underline{\text{C}}$ 7. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 2$.

$y = \frac{1}{2}x^2,\ y = 2, x = 0$




a. $\frac{8}{15}\pi$

b. $\frac{32}{15}\pi$

$\boxed{\text{c. } \frac{64}{15}\pi}$

d. $\frac{4}{15}\pi$

e. $\frac{16}{15}\pi$

$$\int_0^2 \pi\left(2 - \frac{1}{2}x^2\right)^2 dx$$

$$= \int_0^2 \pi\left(4 - 2x^2 + \frac{1}{4}x^4\right) dx$$

$$= 4\pi x - \frac{2\pi}{3}x^3 + \frac{\pi}{20}x^5 \Big|_0^2$$

$$= 8\pi \cdot \frac{15}{15} - \frac{16\pi}{3} \cdot \frac{5}{5} + \frac{\cancel{16}\ 8\ \cancel{32\pi}}{\cancel{20}\ \cancel{10}\ 5} \cdot \frac{3}{3}$$

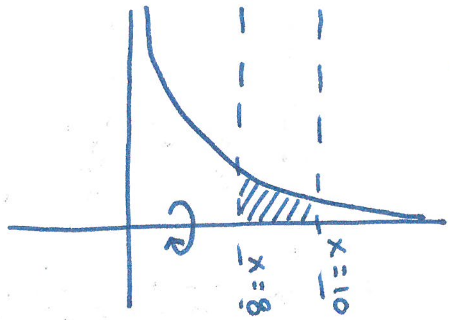
$$= \frac{120\pi}{15} - \frac{80\pi}{15} + \frac{24\pi}{15}$$

$$= \frac{64\pi}{15}$$

Name: ____________________ ID: A

D 8. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the $x$-axis.

$y = \frac{1}{x}$, $y = 0$, $x = 8$, $x = 10$




$\int_8^{10} \pi\left(\frac{1}{x}\right)^2 dx$

$= \int_8^{10} \pi x^{-2} dx$

$= -\frac{\pi}{\cancel{2}x^{\cancel{2}}}\Big|_8^{10}$

$= \frac{-\pi}{\cancel{200}_{25\cdot 8}} - \left(\frac{-\pi}{\cancel{128}_{2\cdot 8\cdot 8}}\right)$ $\quad \frac{4}{4}\cdot\frac{-\pi}{10} - \left(\frac{-\pi}{8}\right)\cdot\frac{5}{5}$

$= \frac{-16\pi + \cancel{25}\pi}{3\cancel{200}} = \frac{9\cancel{\pi}}{\cancel{3}200}$ $\quad \frac{-4\pi}{40} + \frac{5\pi}{40} = \frac{\pi}{40}$

a. $\frac{13}{80}\pi$

b. $\frac{9}{40}\pi$

c. $\frac{9}{80}\pi$

d. $\boxed{\frac{1}{40}\pi}$

e. $\frac{1}{80}\pi$

C 9. Find the arc length of the graph of the function $y = \frac{x^3}{6} + \frac{1}{2x}$ over the interval [1,2].

$= \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$

$y' = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$

a. $\frac{15}{4}$

b. $\frac{49}{12}$

c. $\boxed{\frac{17}{12}}$

d. $\frac{15}{8}$

e. $\frac{49}{24}$

$\int_1^2 \sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}\, dx = \int_1^2 \sqrt{1 + \frac{1}{4}x^4 - \frac{1}{2} + \frac{1}{4x^4}}\, dx$

$= \int_1^2 \sqrt{\frac{1}{2} + \frac{x^4}{4} + \frac{1}{4x^4}}\, dx = \int_1^2 \sqrt{\frac{1}{4x^4}\left(2x^4 + x^8 + 1\right)}\, dx$

$= \int_1^2 \frac{1}{2x^2}\sqrt{x^8 + 2x^4 + 1}\, dx = \int_1^2 \frac{1}{2x^2}\sqrt{(x^4+1)^2}\, dx$

$= \int_1^2 \frac{1}{2x^2}(x^4 + 1)\, dx = \int_1^2 \left(\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right) dx =$

$= \frac{1}{6}x^3 - \frac{1}{2}x^{-1}\Big|_1^2 = \left(\frac{\cancel{8}^{4}}{\cancel{6}_{3}} - \frac{1}{4}\right) - \left(\frac{1}{6} - \frac{1}{2}\right)$



$= \frac{16}{12} - \frac{3}{12} - \frac{2}{12} + \frac{6}{12} = \frac{17}{12}$

Name: ____________________ ID: A

D 10. Find the area of the surface generated by revolving the curve about the $x$-axis.

$y = 4\sqrt{x}, 1 \le x \le 7.$

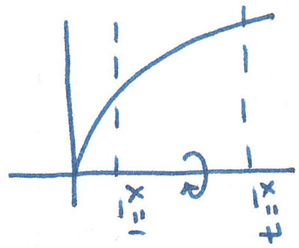
a. $\frac{1}{3}\left(44^{\frac{3}{2}} - 20^{\frac{3}{2}}\right)\pi$

b. $\frac{2}{3}\left(32^{\frac{3}{2}} - 8^{\frac{3}{2}}\right)\pi$

c. $\frac{1}{3}\left(32^{\frac{3}{2}} - 8^{\frac{3}{2}}\right)\pi$

d. $\frac{2}{3}\left(44^{\frac{3}{2}} - 20^{\frac{3}{2}}\right)\pi$

e. $\frac{1}{6}\left(32^{\frac{3}{2}} - 8^{\frac{3}{2}}\right)\pi$




$y = 4x^{1/2}$

$y' = 2x^{-1/2} = \frac{2}{\sqrt{x}}$

$\int_1^7 2\pi (4\sqrt{x})\sqrt{1 + \left(\frac{2}{\sqrt{x}}\right)^2}\, dx$

$= \int_1^7 8\pi \sqrt{x}\sqrt{1 + \frac{4}{x}}\, dx = 8\pi \int_1^7 \sqrt{x+4}\, dx$

$u = x + 4$

$du = dx$

$4\pi \int_{x=1}^7 u^{1/2}\, du = 4\pi \cdot \frac{2}{3}(x+4)^{3/2}\Big|_1^7$

$= \frac{8\pi}{3}\left(11^{3/2} - 5^{3/2}\right)$

$= \frac{2\pi}{3} \cdot 2^3 \left(\sqrt{11^3} - \sqrt{5^3}\right)$

$\left(\sqrt{44}\right)^3 - \left(\sqrt{20}\right)^3$

$\left(\sqrt{4}\sqrt{11}\right)^3 - \left(\sqrt{4}\sqrt{5}\right)^3$

$8 \cdot 11^{3/2} - 8 \cdot 5^{3/2} \quad \cdot \frac{2\pi}{3}$