- 8.1 #5-45 odd basic integration techniques
- 8.2 #1-29 odd integration by parts

$$\int u\,dv = uv - \int v\,du$$

15. $\int t\ln(t+1)\,dt = \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \frac{t^2}{t+1}\,dt$

$u = \ln(t+1)$ $\quad dv = t\,dt$ $\quad u = t+1 \to u-1 = t$

$du = \frac{dt}{t+1}$ $\quad v = \frac{1}{2}t^2$ $\quad du = dt \quad t^2 = (u-1)^2$

$$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \frac{(u-1)^2}{u}\,du$$

$$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \frac{u^2-2u+1}{u}\,du$$

$$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \left(u - 2 + \frac{1}{u}\right)du$$

$$= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\left(\frac{1}{2}u^2 - 2u + \ln|u|\right) + C$$

$$= \boxed{\frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\left(\frac{1}{2}(t+1)^2 - 2(t+1) + \ln|t+1|\right) + C}$$

19. $\int \frac{xe^{2x}}{(2x+1)^2}\,dx$

$u = xe^{2x}$

$du = (1 \cdot e^{2x} + x(2e^{2x}))\,dx$

$du = e^{2x}(2x+1)\,dx$

$\int dv = \int \frac{dx}{(2x+1)^2}$

$u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$V = \int \frac{1}{2}\frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du$

$V = -\frac{1}{2}u^{-1} = -\frac{1}{2u}$

$V = -\frac{1}{2(2x+1)}$

$= \frac{-xe^{2x}}{2(2x+1)} - \int -\frac{1}{2\cancel{(2x+1)}} \cdot e^{2x}\cancel{(2x+1)}\,dx$

$= \frac{-xe^{2x}}{2(2x+1)} + \frac{1}{2}\int e^{2x}\,dx$

$= \boxed{\frac{-xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C}$

67. $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u\,du = \frac{1}{2}u^2 = \boxed{\frac{1}{2}(\ln x)^2 + C}$

68. $\int x \ln x\,dx$

$u = \ln x$ $\quad dv = x\,dx$

$du = \frac{1}{x}\,dx$ $\quad V = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx$

$= \boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

58. $\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$= x\tan x - \int \frac{\sin x\,dx}{\cos x}$

$u = \cos x$

$du = -\sin x\,dx$

$= x\tan x + \int \frac{du}{u} = x\tan x + \ln|u| = x\tan x + \ln|\cos x| \Big|_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4}\cdot 1 + \ln\frac{\sqrt{2}}{2} - 0$

$= \boxed{\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}}$

Solve the differential equation.

40. $\frac{dy}{dx} = x^2\sqrt{x-1}$

$u = x-1 \rightarrow u+1 = x$

$du = dx$

$x^2 = (u+1)^2$

$\int dy = \int x^2\sqrt{x-1}\,dx$

$y = \int (u+1)^2\sqrt{u}\,du = \int (u^2+2u+1)u^{1/2}\,du$

$y = \int (u^{5/2} + 2u^{3/2} + u^{1/2})\,du$

$y = \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$y = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

4. $\int \underline{\cos^3 x} \sin^4 x \, dx = \int \sin^4 x \, \underline{\cos^2 x} \cos x \, dx$

$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^4 x - \sin^6 x) \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int (u^4 - u^6) \, du = \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$