- 8.1 #5-45 odd — basic integration techniques
- 8.2 #1-29 odd — integration by parts $\int u\,dv = uv - \int v\,du$
- 8.3 #1-11odd; 19-31odd; 47-63odd — trigonometric integrals
- 8.4 #5-15odd; 21-39odd — trigonometric substitution
- 8.5 #11-21 odd — partial fractions

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Important things coming up:

Test 3, Part B Retest (Volume of solids of revolution) - Fri. Feb 3

Test 3, Part A Retest (Area & arc length) - see me to schedule

Take-home Quiz due Wed. Feb 8

Test 4 (integration techniques) - Thurs. Feb 9

Final Exam - Thurs. Feb 16

# 8.5 Partial Fractions

$$\int \frac{1}{x^2-5x+6}\,dx = \int\left(\frac{1}{x-3} + \frac{-1}{x-2}\right)dx = \boxed{\ln|x-3| - \ln|x-2| + C}$$

$$\frac{1}{\underset{(x-3)(x-2)}{x^2-5x+6}} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$\frac{A}{x-3}\cdot\frac{x-2}{x-2} + \frac{B}{x-2}\cdot\frac{x-3}{x-3}$$

$$= \frac{Ax-2A+Bx-3B}{(x-3)(x-2)} = \frac{(A+B)x+(-2A-3B)}{(x-3)(x-2)}$$

$$\begin{cases} A+B=0 \\ -2A-3B=1 \end{cases} \Rightarrow \begin{array}{r} 2A+2B=0 \\ -2A-3B=1 \\ \hline -B=1 \end{array} \qquad \begin{array}{l} B=-1 \\ A=-B=-(-1)=1 \end{array}$$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

$x(x^2+2x+1)$

$x(x+1)(x+1)$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$= \frac{A}{x}\cdot\frac{\overset{x^2+2x+1}{(x+1)^2}}{(x+1)^2} + \frac{B}{x+1}\cdot\frac{\overset{x^2+x}{x(x+1)}}{x(x+1)} + \frac{C}{(x+1)^2}\cdot\frac{x}{x}$$

$$= \frac{Ax^2+2Ax+A+Bx^2+Bx+Cx}{x(x+1)^2}$$

$$= \frac{(A+B)x^2+(2A+B+C)x+A}{x(x+1)^2}$$

$A+B=5$ $\quad B=5-A=5-6=\boxed{-1=B}$

$2A+B+C=20$

$\boxed{A=6}$ ✓ $\quad C=20-2A-B$

$\boxed{C=9}$ $\quad =20-12+1=9$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx = \int\left(\frac{6}{x}+\frac{-1}{x+1}+\frac{9}{(x+1)^2}\right)dx$$

$u=x+1$

$du=dx$

$9u^{-2}du$

$$=\boxed{6\ln|x|-\ln|x+1|-\frac{9}{x+1}+C}$$

$$\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}\,dx$$

$$\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{A}{x}+\frac{B}{x-1}+\frac{Cx+D}{x^2+4}$$

$$\frac{A}{x}\cdot\frac{\overset{x^3-x^2+4x-4}{(x-1)(x^2+4)}}{(x-1)(x^2+4)} + \frac{B}{x-1}\cdot\frac{\overset{x^3+4x}{x(x^2+4)}}{x(x^2+4)} + \frac{(Cx+D)}{x^2+4}\cdot\frac{\overset{x^2-x}{x(x-1)}}{x(x-1)}$$

$$\frac{Ax^3-Ax^2+4Ax-4A+Bx^3+4Bx+Cx^3+Dx^2-Cx^2-Dx}{x(x-1)(x^2+4)}$$

$$=\frac{\overset{2}{(A+B+C)}x^3+\overset{0}{(-A+D-C)}x^2+\overset{-4}{(4A+4B-D)}x\overset{-8}{-4A}}{x(x-1)(x^2+4)}$$

$C=2$ $\quad D=4$

$A+B+C=2$

$-A-C+D=0$

$4A+4B-D=-4$

$-4A=-8 \quad \Rightarrow A=2$

$B+C=0$

$D-C=2$ $\Big\}$ $B+D=2$

$4B-D=-12$

$5B=-10$

$B=-2$

$$\int\left(\frac{2}{x}+\frac{-2}{x-1}+\frac{2x+4}{x^2+4}\right)dx$$

$$=2\ln|x|-2\ln|x-1|+\int\frac{2x+4}{x^2+4}\,dx$$

$$=2\ln|x|-2\ln|x-1|+\int\frac{2x}{x^2+4}\,dx+\int\frac{4\,dx}{x^2+4}$$

$u=x^2+4$

$du=2x\,dx$

$$=2\ln|x|-2\ln|x-1|+\ln(x^2+4)+\int\frac{4\,dx}{x^2+4}$$

$$=2\ln|x|-2\ln|x-1|+\ln(x^2+4)+4\cdot\frac{1}{2}\arctan\frac{x}{2}+C$$

$$=\boxed{2\ln|x|-2\ln|x-1|+\ln(x^2+4)+2\arctan\frac{x}{2}+C}$$

44. $\displaystyle\int \frac{\sec^2 x}{\tan x(\tan x+1)}\,dx = \int \frac{du}{u(u+1)}$

Let $u = \tan x$
$du = \sec^2 x$

$\dfrac{1}{u(u+1)} = \dfrac{A}{u} + \dfrac{B}{u+1}$

$= \dfrac{Au + A + Bu}{u(u+1)}$

$A+B=0$
$A=1, B=-1$

$= \displaystyle\int \left(\frac{1}{u} - \frac{1}{u+1}\right) du$

$= \ln|u| - \ln|u+1| + C$

$= \boxed{\ln|\tan x| - \ln|\tan x+1| + C}$

Find the Partial Fraction Decomposition

6. $\dfrac{2x-1}{x(x^2+1)^2}$

$= \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$

$= \dfrac{A}{x}\cdot\dfrac{(x^2+1)^2}{(x^2+1)^2} + \dfrac{Bx+C}{x^2+1}\cdot\dfrac{x(x^2+1)}{x(x^2+1)} + \dfrac{Dx+E}{(x^2+1)^2}\cdot\dfrac{x}{x}$