- 6.3 #15, 17, 19
- 7.5 #5, 6,7,37,38

solving differential equations via separation of variables
work

6.3 Solving Differential Equations by Separation of Variables

$y = 5x^3 - \cos x$

what is the differential of y?

$dy = (15x^2 + \sin x)\,dx$

$\frac{dy}{dx} = 15x^2 + \sin x$

$y' = \frac{dy}{dx} = \frac{d}{dx}(y)$

~~5.7~~ Ex 3 - Find the general solution.

$$(x^2+4)\frac{dy}{dx} = xy$$

$$\frac{(x^2+4)\,dy}{y(x^2+4)} = \frac{xy\,dx}{y(x^2+4)}$$

$$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2+4}$$

$$\ln|y| = \left(\frac{1}{2}\ln|x^2+4| + C\right)$$

$$e^{\ln|y|} = e^{\ln\sqrt{x^2+4}}\,e^{C}$$

$$|y| = e^{\ln\sqrt{x^2+4}}\,e^{C}$$

$$y = \pm A\sqrt{x^2+4}$$

Ex 4 Find a particular solution.

$$xy\,dx + e^{-x^2}(y^2-1)\,dy = 0 \quad ; \; y(0)=1$$

$$xy\,dx = -e^{-x^2}(y^2-1)\,dy$$

$$\int \frac{x\,dx}{-e^{-x^2}} = \int \frac{(y^2-1)\,dy}{y}$$

$$\int -xe^{x^2}\,dx = \int \left(\frac{y^2}{y} - \frac{1}{y}\right)dy$$

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$\int -\frac{1}{2}e^{u}\,du = \int \left(y - \frac{1}{y}\right)dy$$

$$-\frac{1}{2}e^{x^2} = \frac{1}{2}y^2 - \ln|y| + C \qquad y(0)=1$$

$$-\frac{1}{2}e^{0^2} = \frac{1}{2}(1)^2 - \ln|1| + C$$

$$-\frac{1}{2} = \frac{1}{2} - 0 + C \Rightarrow C = -1$$

$$\boxed{-\frac{1}{2}e^{x^2} = \frac{1}{2}y^2 - \ln|y| - 1}$$

~~6.5~~ 7.5

work done by an expanding gas

initial volume: $1\ ft^3$

initial pressure: 500 pounds per ft^2

gas expands to a volume of $2\ ft^3$

Find the work done by the gas.

Assume pressure is inversely proportional to volume.

$P = \frac{K}{V}$ since $500 = \frac{K}{1}$, $K = 500$

$$W = \int_{V_0}^{V_1} \frac{K}{V}\,dV = \int_1^2 \frac{500}{V}\,dV = 500 \ln|v| \Big|_1^2$$

$$= 500 \ln 2 \approx 346.6 \text{ foot-pounds}$$

Compressing a spring

A force of 750 lb compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring additional 3 in.

Hooke's Law: $F(x) = kx \Rightarrow F(x) = 250x$

$750 = k \cdot 3$

$250 = k$

$$W = \int_3^6 250x\,dx = 125x^2 \Big|_3^6 = 125(36) - 125(9)$$

$125(27) = 3375$ inch-pounds