3.9 #8 $y = 1 - 2x^2 = f(x)$; $x = \overset{c}{0}$; $\Delta x = dx = -0.1$

$y' = -4x$

Compare dy and Δy for the given values of x and Δx.

$T(x) \approx f(x)$

$\Delta y = f(x) - f(c)$

$dy = T(x) - f(c)$

$\boldsymbol{\Delta y = f(c + \Delta x) - f(c)}$

$= f(0 + (-0.1)) - f(0)$

$= 1 - 2(-0.1)^2 - (1 - 2(0)^2)$

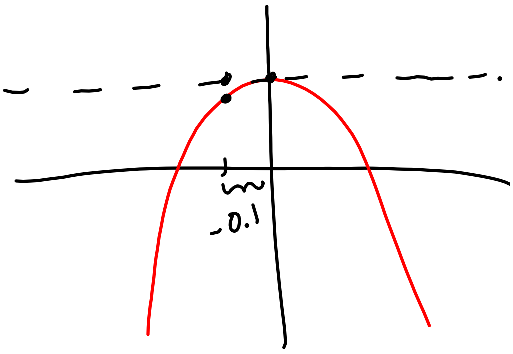
$= 1 - 0.02 - 1$

$\Delta y = -0.02$

$\boldsymbol{dy = f'(c)dx}$

$= -4(0) \cdot (-0.1)$

$dy = 0$



Find the differential dy.

$dy = f'(x)dx$

12. $y = 3x^{2/3}$

$dy = 2x^{-1/3}dx$

16. $y = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$

$dy = \left(\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}\right)dx$

14. $y = \sqrt{9 - x^2}$

$= (9 - x^2)^{1/2}$

$dy = \left(\frac{1}{2}(9 - x^2)^{-1/2}(-2x)\right)dx$

$dy = \frac{-x\,dx}{\sqrt{9 - x^2}}$

20. $y = \frac{\sec^2 x}{x^2 + 1}$

$$y = \frac{\sec^2 x}{x^2+1} = \frac{(\sec x)^2}{x^2+1}$$

$$y' = \frac{(x^2+1)\left[(\sec x)^2\right]' - (\sec x)^2 \cdot \left[x^2+1\right]'}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(2\sec x \cdot \sec x \tan x) - (\sec x)^2 \cdot 2x}{(x^2+1)^2}$$

$$dy = \frac{(x^2+1)(2\sec^2 x \tan x) - 2x\sec^2 x}{(x^2+1)^2}\,dx$$

3.9 #46

Use differentials to approximate $\sqrt[3]{26}$

$$\left.\begin{aligned} \Delta y &= f(c+\Delta x) - f(c) \\ dy &= f'(x)dx \\ \Delta y &\approx dy \end{aligned}\right\} \rightarrow \boldsymbol{f(c+\Delta x) - f(c) \approx f'(x)dx}$$

$$\boxed{f(c+\Delta x) \approx f(c) + f'(c)dx}$$

$f(x) = \sqrt[3]{x} = x^{1/3}$; $c = 27$; $\Delta x = dx = -1$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$\sqrt[3]{26} = \sqrt[3]{27+(-1)} \approx \sqrt[3]{27} + \frac{1}{3(\sqrt[3]{27})^2} \cdot (-1)$

$= 3 + \frac{-1}{27} = \frac{81}{27} - \frac{1}{27} = \frac{80}{27}$

$\frac{80}{27} \approx 2.962$

$\sqrt[3]{26} \approx 2.962$

Recall rules of exponents: $x^{m/n} = (x^m)^{1/n} = \left(x^{1/n}\right)^m$

$= \sqrt[n]{x^m} \quad = \left(\sqrt[n]{x}\right)^m$

3.9 #50

Use differentials to approximate $\tan(0.05)$.

$f(c+\Delta x) \approx f(c) + f'(c)dx$

$f(x) = \tan x$; $c = 0$; $\Delta x = dx = 0.05$

$f'(x) = \sec^2 x$

$$\tan(0+0.05) = \tan 0 + (\sec^2(0))\cdot(0.05)$$

$$= 0 + 1\cdot 0.05$$

$\tan 0.05 \approx 0.05$

calculator yields: 0.050

3.9

#37, 39

#14, 15, 18, 19